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Acquisition of Comparison Concepts in Grades 3 and 6: Findings from Diagnostic Tests of Student Achievement

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Abstract: The aim of the present paper is to explore how students of grades 3 and 6 learn basic comparison concepts ("so much more/less", "so many times more/less", "How much more/less?", "How many times more/less?") and master the skill of using them in order to solve practical problems in line with the requirements of National Standard for Basic Education. Standard mathematical competence of Latvian students in the National Standard for Basic Education is stipulated in three stages of learning: grades 1-3, grades 4-6, grades 7-9. In 2013/2014, National Centre for Education (NCE) of the Republic of Latvia conducted national diagnostic tests preceded by a pilot research with a view to preparing methodological suggestions for teachers on how to analyze the results of the diagnostic test and how to apply them to help each individual student reach high achievement by the end of the academic year. Selection of problem tasks for the diagnostic test and data analysis were conducted in line with the OECD Programme for International Student Assessment (PISA). The present paper is aimed at analyzing the results of diagnostic tests, the errors made by students, their causes and methodological solutions with a view to facilitating students' acquisition of basic comparison concepts in mathematics.

Key words: Comparison concepts, skills, acquisition, student academic achievement, diagnostic tests, criteria for evaluation, diagnostic map.

1. Introduction

In modern learning, learning outcomes belong in the new learning paradigm of the European Union [1]. Formulation of learning outcomes in the learning process allows for facilitating the formation of students' experience instead of laying emphasis on the curriculum. Learning outcomes are essential when developing national qualification frame structures. It is also important that they match the basic standpoints of our national education for 2014-2020 [2] and sensitize students to education for sustainable development [3-4].

Recent international research argues that every person needs mathematical competence to deal with problems of daily life and that mathematical competence is an inherent component of many disciplines, professions and realms of human activity [5]. To develop this competence, students need to master certain basic skills already in elementary school. Finding the means towards this end has been a lifelong concern for Jānis Mencis Sr. (1914-2011)—an outstanding Latvian educator, author of numerous text books and creator of a methodological system of learning mathematics. The present research also endeavors to contribute to this topical problem. According to Mencis Sr., apart from mastering purely mathematical skills and abilities, learning mathematics also involves developing general intellectual skills and abilities. That is to say, before finding solution to a given problem, one is capable of drawing up an action plan, focusing one's attention, critically examining one's performance, exerting self-control over the outcome of one's work, etc. [6, p. 12]. Mencis Sr. analyzes the following hyperbolized statement by

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Western methodologists: “Know-how without understanding is ten times more valuable than understanding without know-how” [6, p. 13]. Mencis Sr. was concerned that learners seem to know much but lack that many skills. His ideas remain topical to this day.

Findings from the OECD international study in 2012 also enhance the topicality of the present research.

The Programme for International Student Assessment (PISA) construes competences as knowledge and skills necessary for future life. PISA defines mathematical competence [7-8] as follows:

(1) skill of identifying, determining, and understanding the role and place of mathematics in the world;

(2) skill of making well-justified decisions;

(3) ability to use mathematics to meet one’s life needs as a constructive, interested and reasonable citizen.

This definition emphasizes the role of mathematics as an academic subject that, being taught at school, lays special emphasis on the processes related to problem-solving in the context of real life, by analyzing them mathematically, using adequate mathematical knowledge and assessing the solution in the context of a given problem.

How do Latvian students appear in comparative international studies in mathematics?

“The relative number of elementary school students with a low competence level in mathematics, natural sciences and reading in Latvia is smaller than in OECD countries on average, which is certainly positive; however, compared to OECD countries, Latvia also has fewer students with a high competence in mathematics, natural sciences and reading” [9, p. 67].

Final examination papers of grade 9 students over the last two years reveal that the average completion of basic tasks in Part 1 of the exam is plummeting and approaches the critical level [10].

How to deal with the problem of low academic achievement and enhance student motivation for learning mathematics? One efficient means that was used in Latvia more than 20 years ago to improve the state of mathematics education was teacher- and school-conducted diagnostic tests as well as national tests. In a Soviet school, so high were the requirements set for mathematics that only a small fraction of students were able to meet them. In the time period from 1986-1992, national-level diagnostic tests were used to reform the state-determined mathematics curriculum and learning outcomes. In the first years of the independent Latvia in 1991/1992, the authors of the present paper along with co-authors drafted the first project of the standard in mathematics for grades 1-9 [11]. The core ideas of this draft standard, “Requirements for students of grades 1-9 in mathematics” are still being relied upon when drafting new academic standards and programs. In Latvian mathematics education, a dynamic system of standards has been developed and is still being improved upon. The latest version of the standard for mathematics was published in the regulation of the Cabinet of Ministers of the Republic of Latvia [MK 12.08.2014. Nr.468. (Appendix *6. Mathematics. Standard of Teaching Mathematic for Grades 1-9*)].

National tests measure the results against the standard requirements for student achievement at certain stages of learning and, vice versa, analysis of test results helps adjust standard requirements for learning outcomes. This idea is reiterated in EU documents: “In standards-based systems, which are increasingly common across countries, governments set standards for student attainment, clearly defining the knowledge and skills students are expected to attain at different stages of their education. The curriculum covers the objectives identified in standards, and student assessments focus on attainment of standards. The core logic of standards-based systems rests upon the alignment of these key elements. If the assessments do not well match the curriculum and the standards, then results have little value in judging how well the students are learning and in diagnosing school or student needs” [12, p. 3].

The first diagnostic test organized by the Ministry of Education, “Test of student calculation skills and abilities” was broadcast on Latvian Television on January 21, 1986, testing grades 6, 8, and 11 (approximately 45,000 students). On May 6, 1986, problems were offered for a repeated test of student calculation skills and abilities in the same grades. From 1986 to 1994, key points for activating interior control at school were investigated and planned. Methodological aid was provided to teachers of mathematics to determine the essential issues in the curriculum and organize diagnostic tests before each repeated reading of the course in subsequent classes. This purposefully directed the teachers towards the learning outcomes to be attained at the end of elementary school. In those years, by activating interior school controls, a positive increase was witnessed in many measurements of the level of student of attainment, from the deficient and critical to the sufficient and optimal one.

In the academic year 2013/14, to facilitate acquisition of the standard requirements in mathematics, the National Centre for Education in accordance with the regulations of the Cabinet of Ministers organized national diagnostic tests in grades 3 and 6.

In the academic year 2014/15, national tests were conducted in all learning stages: in grades 3, 6 and 8 before summative revision work and in grade 9 as a final examination in mathematics. This study focuses on one mathematical conceptual group (comparison concepts) which is incorporated in the curricula of each grade and, with an increasing level of difficulty, is tested in all tests.

The paper explores the aims and functions of diagnostic tests, the designing of test instruments [13] to determine each student’s level of basic skills and general intellectual operations in line with the requirements of Basic Education Standard in mathematics. The paper also considers the means of determining the level of academic achievement that characterizes the entire class in general and making necessary adjustments in the learning process according to the diagnostic test results.

Further, the authors analyze the current situation in researching student academic achievement.

2. Research Methodology

The research comprises statistical analysis of the results from national diagnostic tests in mathematics for grades 3 (n = 16,767) and 6 (n = 16,746), conducted in 2014. These are compared against statistical data from final tests and examinations in mathematics in the academic years 2012/13 and 2013/2014. In addition, content analysis of student tests from 8 schools and 13 classes (n = 372) is performed.

In the academic year 2013/2014, test 1 (n = 170) and tests 2, 3, 4 (n = 52) were administered within a pilot research to grade 6. Test 1 covered *Actions with rational numbers*; test 2—*Basic notions of comparison*, test 3—*Per cent problems*, and test 4—*Basic elements of geometry*. The content and data analysis of the tests is incorporated in methodological recommendations for teachers, students and their parents in preparation for the national diagnostic test in grade 6 [14].

To some extent, a purposively organized system of testing aligns with action research [3-4], wherein a teacher plans the learning outcomes and their attainment, involves students in the learning process through which they acquire certain knowledge, skills and attitudes, and reflects on the results.

An essential feature of action research is initiating reflection. Learning outcomes are reflected in tests. Subsequent to analysis of these tests, individual consultations are planned with specific suggestions on how to improve deficient learning outcomes. In this way, students cooperate with teacher to gradually acquire the experience of deliberate learning.

Over the course of the present research, the authors also studied textbooks in mathematics and methodological aids for teachers of grades 1-6, which were relied upon to formulate methodological recommendations for teachers.

3. Theoretical Background

3.1. Learning Outcomes and Opportunities for Their Improvement in the Learning Process

According to the European Commission regulations, "outcomes of learning are statements concerning what a learner knows, understands, and is able to do having completed learning" [1]. This presupposes that learning outcomes are impacted not only by the learning process as such, but also the student's motivation to learn, the teacher's success in directing this process and the way the achieved results are evaluated. Five competence spheres are distinguished in mathematics:

(1) learning basic skills and operations;

(2) understanding mathematical concepts and principles;

(3) using mathematics in everyday life;

(4) discussion of mathematics;

(5) mathematical reasoning [5, p. 131].

This direction was pursued by didacts of mathematics who created methodological aids [15-18]. Organization of a contemporary learning process as well as planning and evaluation of the learning outcomes rests on the tenets of psychology about the child's cognitive development—Piaget [19-21], zone of proximal development—Vygotsky [22], developmental education and development of thinking in mathematics—Davydov [23], constructivist beliefs about active learning—Bruner [24], critical thinking [25-26] etc.

The present research is more concerned with investigating the two former spheres in diagnostic tests. To facilitate improvement of learning outcomes and develop methodological recommendations for teachers, opinions of students and teachers alike were canvassed [27-28]. The use of everyday life situations in mathematical problems came into focus in order to relate learning of mathematics to students' experience, also non-standard problem samples were offered for students' independent work.

3.2. Diagnostic Tests and Educational Diagnostics

Research proves that "evaluation is too often used to grade students instead of helping them improve their achievement. To enable the development of knowledge and skills, such forms of evaluation must be more widely used that contain feedback information and thus make it possible to identify and solve problems" [5, p. 65]. This confirms the need for shifting of priorities in evaluation, purposeful reflexive activity and self-evaluation which allows for achieving shared accountability among students for their learning outcomes.

Diagnostic work to determine academic achievement is a facet of educational diagnostics analyzed by Freidenfelds and Ūsiņš [29] who argue that improving academic achievement in each class always involves a complicated set of educational actions for each particular student. Apart from the teacher, other participants in this process are students, parents and, in some cases, the wider community. According to Freidenfelds and Ūsiņš, however, the management of the entire system must stay in the professional hands of a teacher. The authors note that the first stage in this goal-oriented process is diagnostics of academic achievement which entails determining student knowledge, skills, experience of creative action and general intellectual actions (for an individual student, a group of students, the entire class), the second stage—diagnostics-based implementation of an action program, the third stage—assessment of the work carried out to improve academic achievement and setting further tasks depending on the outcome.

3.3. Functions of a Diagnostic Test

Review of relevant theoretical literature and normative requirements suggests that:

(1) Diagnostic tests must provide information about the student's actual knowledge, modes of action (skills), skills of learning, and cognitive action;

(2) The procedure of diagnostic testing must provide for the validity and reliability of the acquired data;

(3) Diagnostic tests must take place at a time when the learning process is still controllable;

(4) Analysis of students' tests must be used to apprehend the level of knowledge and skills in each student and across the entire class in keeping with the standard of any given academic subject;

(5) Diagnostic tasks must include questions that are more challenging and necessary for further stages of learning.

3.4. The Aims of Elaborating Diagnostic Tests

Diagnostic tests are a form of tests that may be used for several purposes at once.

"The results of national tests are used, firstly, with regard to individual students, secondly, with regard to schools and local authorities, and, finally, in relation to the education system as a whole" [30, p. 49].

In the academic year 2013/2014, more attention was paid to the first group—diagnostics of student achievement.

Diagnostics of student achievement aim to:

(1) elaborate the instruments for determining every learner's progress in terms of basic skills and general means of intellectual activity in keeping with Basic Education Standard [31];

(2) determine the overall level of achievement in the entire class and adjust the learning process in keeping with the outcomes of diagnostic test;

(3) design methods for studying the dynamics of academic achievement in respective grades.

Yet, the results of this diagnostic test should not be compared across schools and classes. Comparing results across schools and classes is a very complex procedure. This may be done only after "provision of a unified monitoring of the quality of education on the national level. By 2017, 50% of educational establishments are to be included in this monitoring, reaching 100% by 2020" [2, p. 118].

At the initial stage, it is more important in educational diagnostics to analyze the indicators of skill acquisition, locate causes of errors, select the educational and methodological approaches for eradicating these shortcomings in order to help every student improve their mathematical competence (attain individually high achievement), as well as to pinpoint the dynamics between diagnostic tests and repeated check-ups.

3.5. The Choice of Tasks for the Diagnostic Test

First diagnostic tests in Latvia included simple tasks, which made it possible to diagnose the causes for student academic achievement and failure. Also, these tasks helped study the dynamics of student knowledge and skills. Hence, if 20% of grade 6 students cannot calculate the value of the numerical sequence $5 – 8 + 6$, it may be safely assumed that at least 20% of students will not manage the simplification of the algebraic expression $5a – 8a + 6a – 4$, etc.

Nowadays, in line with Latvian Basic Education Standard requirements and those implemented in international research, diagnostic tests include mathematical problems the context of which depicts the situations that students could encounter in real life.

The OECD PISA basic strategy for solving mathematical problems which was used in producing pilot research tests for grade 6 is as follows:

"real life situation (context) >> to identify the mathematical problem and transform the context to match it >> to solve the mathematical problem >> to express the acquired mathematical result in accordance with the context" [8, p. 30].

The skills needed for solving PISA problems were divided into three groups:

(1) reproduction problems;

(2) interconnection problems;

(3) mathematical thinking and generalization problems [8].

Tests ought to comprise problems of all levels: reproduction of the acquired skills, applying the skills in similar situations to solve interconnection problems, transferring the knowledge to a new, non-standard situation to solve generalization problems.

For instance, some problems from test 2 of the pilot research (problems of test 2 of the pilot research [14, pp. 14-26] contained notions "so much more/less", "so many times more/less", "How much more/less?", "How many times more/less?" and their practical application in solving problems):

Problem 1 (1 point). A packet of "Fairy" biscuits cost 1.67 € in a shop. In the holiday season its price was reduced by 0.46 €. How much did the packet of biscuits cost after price reduction?

A 1.21 € B 1.98 € C 2.13 € D 0.21 €

Solution of the problem: **98%** (optimal level).

In this problem, students need to understand the notion "so much less", make a mental calculation or do it in a written form, compare the acquired result with the multiple choice options and choose the correct answer. The chosen answer reflects the student's reasoning, or it may also happen to be random. The problem belongs to the group of skills reproduction.

Problem 8 (2 points). Riga is the one of the largest capital cities of northern Europe. In early 2013, it had 643.6 thousand residents. Oslo, the capital city of Norway, had 30.3 thousand fewer residents than Riga. What was the number of residents in Oslo in early 2013? Answer: ….

Solution of the problem: **83%** of students work with the text well, appropriately use the notion "So much more/less", and do correct calculations with decimal fractions. **64%** of students find it difficult to provide the answer; they suggest that the number of residents in Oslo is only a few hundred or that fractions of people live there (missing the word "thousand" in the text). They find it difficult to match the acquired data with the reality, or critically evaluate their solution by asking themselves, "Can it be so?"

The problem belongs to the group of skills interconnection.

Problem 10 (4 points). According to EUROSTAT data (2nd half of 2012), electricity prices for households per 100 kilowatt hours, excluding VAT, were as follows: in Latvia—11.32 €, in Germany—22.49 €, in Croatia—11.07 €, in Romania—8.12 €, and in Bulgaria —7.96 €, while the average price in the European Union was 16.85 €.

(a) How much less was the electricity price in Bulgaria compared to Latvia?

(b) How much more was the electricity price in Germany compared to Latvia?

(c) Compare electricity price in Germany to the average price in the European Union!

How much more or less is it? Justify your answer with a calculation! Answer: ….

Solution of the problem: Students use appropriate means to solve the problems by using numeric models: they find the necessary quantities in the text and use the notion "so much more" and "so much less" in calculations with decimal fractions:

In 10.a—81%, 10.b—93%, in 10.c—81% find the quantities and use them in calculations "How much more or less?", while 64% give the correct answer.

In problem 10.c students encountered some difficulty with formulating the answer to the question, "How much more or less?" Students often cannot formulate a logical and correct answer that was the diagnostic aim of this problem. Formulation of a general judgment is a higher level skill than those included in 10.a and 10.b. Problem 10.c belongs to the group of skills interconnection that is closely related to the group of mathematical reasoning and generalization.

4. Analysis and Discussion of Research Results

Over the course of the present study, acquisition of certain basic skills in grades 3 and 6 was analyzed comparatively. The following questions were addressed: What are the results of test completion in 2013 and 2014 for grades 3, 6, 9? What methodological solutions for helping students learn basic skills are suggested by textbook authors? How to design evaluation criteria and a diagnostic map?

A core issue in elementary school is acquisition of comparison concepts "so much more/less", "so many times more/less", "How much more/less?", "How many times more/less?" and their application in solving practical mathematical problems.

These concepts were included in **problem 4** of the national diagnostic test in grade 3 (7 points):

Draw a broken line ABCD that consists of three line segments! The length of the first line segment is 6 cm, but the second is 2 times shorter than the first one, while the third is by 2 cm longer than the second. Calculate the length of the broken line!

Altogether the problem was solved correctly by 76.44% of students [32].

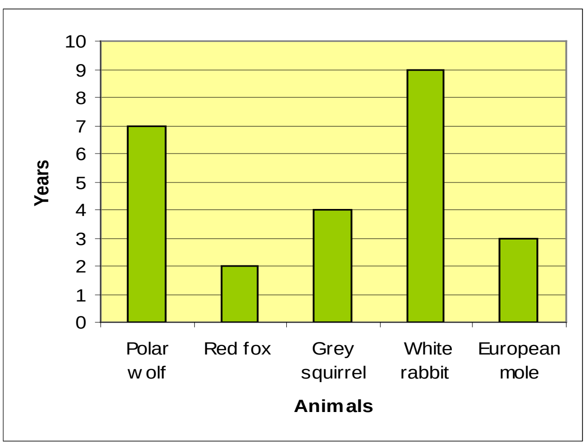
The notion "2 times shorter" is used correctly by 81% of students. In schools with Russian language of instruction the results are better (84%), while in schools with Latvian and Polish as languages of instruction 76% and 67% of students respectively solved the problem.

The notion "by 2 cm longer" is correctly used by 75% of students, respectively by 78% in schools with Russian language of instruction and by 71% in schools with Latvian and Polish languages of instruction. Some students find it difficult to perceive the concepts "longer", "shorter" if they are not specified as "more" or "less". The results also depend on how often these concepts are used and whether recent revisions have been made.

However, in non-standard situations this concept is used correctly only by 30% of students in grade 3. In this case, students faced greater difficulty with the non-standard problem solution strategy "Guess-check" which required summarizing data from a table. It means that having learnt the concepts in itself does not guarantee a high result. It is crucial to master thinking strategies that enable transfer of knowledge to new situations. To investigate the use of the aforementioned concepts in grade 6, two similar problems were analyzed in the pilot research test 2 and the national diagnostic test in grade 6.

For instance, task 9 in grade 6 of the 2nd pilot test (after each problem, its solution in % is indicated):

The column diagram shows the approximate animal life expectancy in wilderness expressed in years (see Figure 1):



(a) How long does the White rabbit live in wilderness? (95%)

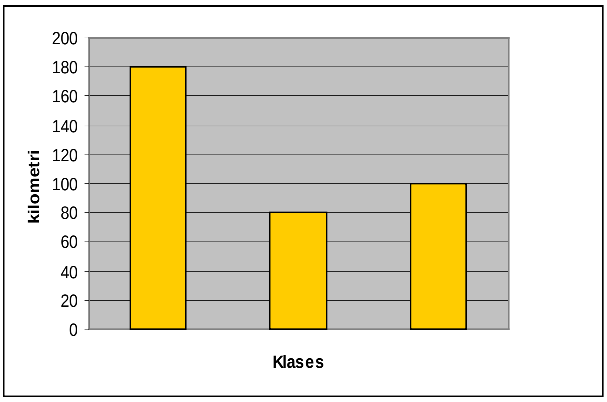
(b) How much shorter is the life expectancy of the Red fox compared to that of the White rabbit? (83%)

(c) How many times longer is the life expectancy of the Polar wolf compared to that of the Red fox? (54%)

(d) Which animals live in wilderness less than 7 years? (88%)

(e) In alphabetical order, list the animals whose life expectancy in wilderness is less than that of the Polar wolf! (86%)

Fig. 1 Diagram for problem 9.



(a) Under each column mark the name of the respective grade (96 %).

(b) How long was the route in km covered by Grade 6b? (95%).

(c) For how many km did Grade 6a covered less compared to 6c? (92%)

(d) How many times is the route covered by Grade 6b as compared with Grade 6a? (62.5%)

(e) How many km on average did each grade cover? (58%)

Fig. 2 Diagram for problem 6.

A more detailed attention is paid to this skill in national testing for grade 6: http://visc.gov.lv/vispizglitiba/eksameni/dokumenti/uzdevumi/2014/6klase/6kl_mat_lv.pdf.

For instance, problem 6 in the national diagnostic test (after each problem, its solution in % is indicated):

The diagram (see Figure 2) shows the routes covered by Grades 6a, 6b, and 6c of the same school. The names of grades are not marked in the diagram. The longest route was covered by Grade 6b. The shortest was covered by the 6a.

Comparison of the solution to problem 9c in pilot research test 2 (n = 52) to that of problem 6d (n = 16 746) yields the following observations:

(a) In pilot research, problem 9c was evaluated with one point, and it does not show how many students fail to understand the use of the ratio “How many times more?” and how many cannot divide 7 by 2;

(b) A similar problem in the national diagnostic test is evaluated with two points, see evaluation criteria in Table 2 that reveal the following:

It is obvious that 28.83% students have received one point for this problem, while 39.13% have received two points. This means that the ratio of two numbers is written (reading the quantities and using them to produce the ratio “How many times more?”) by 28.38% + 39.13% = 67.51% students, while only 39.13% students can divide 180 by 80, where fraction or decimal fraction appears in the quotient. It is possible that more students can do that but they have not written the correct ratio of 180:80.

The diagnostic test defines an essential operation as action that needs to be performed to solve the problem and reflects the aim of the test problem. In this case, the aim of problem 9c was to diagnose how students can use the concept “How many times more?” by writing a two number ratio. The conclusion is that this skill is learnt at a sufficiently high level, which is not far from the critical level. Hence a hypothesis that must be proved: the skill of dividing whole numbers where a fraction or a decimal fraction appears in the quotient has been insufficiently acquired.

In grades 3 and 6, major difficulties emerge in solving problems from the interconnection skills group of mathematical thinking and the generalization skills group. For instance, reading tables or diagrams together with the use of the concept “how many times more/less?” in calculations if the quotient is a fraction or a decimal fraction. As seen from pilot research test 1, “Actions with positive numbers”, students have most difficulties with dividing two whole numbers, especially if zero appears in the middle or the end of the quotient number. Thus, for instance, problem 9 of pilot test 1: *Calculate 151 : 5 and make a mental approximate revision of the calculation.* In December 2013, 47.5% of grade 6 students in various schools solved this problem, while in grades 7 and 8 the percentage of students was 50.25%. This suggests that students have not learnt numeracy and cannot approximately estimate or check the result mentally, without resorting to pen and paper.

The use of the comparison concept is closely related to students' reading competence.

"Students with poor reading literacy tend to find it difficult to learn other subjects and, consequently obtain education as such" [33, p. 71]. While student achievement in mathematics and natural science falls into the scope of average OECD achievement, "Latvian students' reading achievement is still statistically lower than the average OECD level" [9, p. 45]. Reading achievement in 2012 has dropped significantly compared to 2003. This explains the difficulties that students encounter when solving mathematical word problems.

"On the whole, comparing the distribution of the number of students (%) across competence levels suggests that relatively fewer Latvian students can solve tasks of the highest complexity and their number has not changed since 2003" [9, p. 24].

How did students solve a non-standard problem in a diagnostic test in grade 3? That was the 9th and ultimate task:

There are 20 different mushrooms in a basket. The number of king boletes is the smallest. The number of russulas is greater by 4 than that of chanterelles. How many mushrooms of each kind can there be in the basket?

This problem includes concepts such as "least" and "greater by 4". Students are to perceive and assess the numeric ratios of the kinds of mushrooms and combine conditions to find two variants of solution. Both answers were found by 30.64% of students, while one answer by 41.52% of students. This proves that grade 3 students are capable of solving non-standard problems. Only some textbook authors offer problems with elements of combinatorics and illustrate how to reflect the course of solution in a table. The problem belongs to the group of mathematical reasoning and generalization.

In all diagnostic tests offered to teachers in the pilot study, additional problems were included that tested students' creative capability. In every class the creative problem is solved by 1−3 students, which either proves that students are not given enough creative tasks or that there is too little time for solving them.

For instance, the additional task for grade 6 at the end of pilot research test 2:

"Arena Riga" is a multifunctional hall where much space has been provided for sports events. If there were 5 times more spectators than free seats, then the number of occupied seats would exceed that of free seats by 7000. In total, how many seats for sports events are there in "Arena Riga"?

The problem belongs to the group of interconnections that is closely related to that of mathematical reasoning and generalization. The problem includes concepts such as "5 times more" and "exceed by 7000". The problem is solved in grade 6 by using a unit, some students may solve it by introducing an unknown quantity. The problem was fully and correctly solved by 3 students out of 52 in pilot research. To perceive the idea of the problem solution more easily and make a plan of solution, visualization of the problem conditions would be most useful. This was hardly done by any of students in pilot research test 2, but 3 students out of 372 used visualization in the last non-standard problem of the national diagnostic test. Both non-standard problems belong to the group of mathematical reasoning and generalization.

Regarding academic achievement in national diagnostic tests, it appears that the average performance of basic problems in the country in general is fairly good (see Table 1) [10].

In grade 3, the results of the diagnostic test actually reflect the academic results of grade 2, as the students have only just begun to master multiplication and division beyond the table (23 · 4; 72 : 3; 54 : 18).

Table 1 National mathematics test results in elementary school from 2013 and 2014.

Type of test	Average performance
Test by the end of grade 3 in 2013	74.84%
Diagnostic test in grade 3 in 2014	77.54%
Test by the end of grade 6 in 2013	71.65%
Diagnostic test in grade 6 in 2014	68.65%
Examination by the end of grade 9 in 2013	
Part 1	72.80%
Part 2	45.70%
Examination by the end of grade 9 in 2014	
Part 1	66.26%
Part 2	53.14%

How is acquisition of the basic skills of comparison reflected in the final examination at the end of elementary school?

Problem 3. (7 points) in the final examination in **grade 9** in 2012/2013, part 2 [34] (after each problem, its performance in % is indicated).

Column and sector diagrams feature information about Latvia's export.

Produce calculations using the given information!

(a) How many million lats did food export reach in 2010? **(61%)**

In this problem, students need to read the per cent of food export from the sector diagram and the amount of food export in millions of lats from the column diagram. They have to calculate % from a number (basic skill of grade 6).

The task belongs to the group of skills interconnection.

(b) By how many million lats did Latvia's export grow in 2011 as compared to 2009? **(92%)**

This proves optimal mastery of the concept "By how much more?" and its adequate usage to solve practical problems.

(c) By how much % did Latvia's export grow in 2011 as compared to 2009? Round up the result to whole per cent **(22%).**

Students must be able to use the information acquired in the previous problem and know how to calculate per cent growth, they must be able to write the right per cent ratio and calculate it. This problem is of a heightened complexity measured against standard requirements.

In general, it appears that the acquisition level of comparison concepts varies at different stages of elementary school. Learning of mathematical concepts is closely related to learning calculation skills and performing text analysis, which affects the final assessment. Not enough attention is paid to the transfer of knowledge to non-standard situations and development of problem-solving competence among students.

In diagnostic tests, it is not only the choice of corresponding problems that is essential but also the evaluation criteria and following up the results.

4.1. Evaluation of Diagnostic Test Results

The problems for diagnostic tests and their evaluation criteria are designed so that it is possible to determine clearly whether the basic skill envisaged in the standard for mathematics has been acquired or not. The problem is solved **correctly** or **incorrectly** (1 point or 0). After grading the diagnostic test with points, the final assessment of a student's test in points is finalized **by summing up the positive points.**

Thus, for instance, the national diagnostic test for grade 6 on the basic notions of comparison contains problem 6 where the following criteria are used in evaluating student academic achievement (see Table 2).

Table 2 Evaluation criteria for the national diagnostic test (problem 6).

Pr. No.	Criteria corresponding with the requirements of the Standard for mathematics	Total points
6	Use column diagram in solving problems: (a) arrange objects according to size—1p. (b) obtain information from the diagram—1p. (c) compare the value of two rational numbers (read values and use them calculating the difference "By how more/less?") —1p. (d) write a two-number ratio (read quantities and use them for the ratio "How many times more?") —1p. (e) divide whole numbers (quotient is a decimal fraction) —1p. (f) write an expression for calculating the mean value of numbers (understand calculation of the mean value of numbers) —1p. (g) make calculations – 1p.	7 points

Simultaneously with the evaluation of test papers, errors are marked in student worksheets. The best way to indicate student errors is a rather debatable issue in the present epoch of tests. If a teacher indicates the points scored (0, 1, or 2) but does not underline the error, this may be sufficient for a student with high academic achievement, but for a student who faces difficulties in learning mathematics the erroneous place in the solution ought to be shown. The teacher is not to correct the errors, i.e. provide the right solution on the student's worksheet, otherwise acknowledged correction of errors will be missing on the part of the student.

How deep and justified must the student's solutions be? Having analyzed solutions of students from 13 classes of 8 schools, it appears that more correct and precise answers are given if students were asked to produce notes of solution and justification for their actions. Mathematical culture of notes and clarity of solution on the whole yield better results in a class.

Gage and Berliner discuss the advantages and disadvantages of criterion-referenced assessment [35]. In diagnostic tests, the use of problems on the level of reproductive and productive inquiry and diagnosing student achievement via a diagnostic map may drastically counter the disadvantages of criterion-referenced assessment.

4.2. Overview of Test Results with a Diagnostic Map

A diagnostic map is an electronic table for summarizing test results.

The aims for drawing up a diagnostic map are as follows:

(1) to obtain a documented survey of academic a -chievement analysis in the respective class in a giv -en test at a given time;

(2) to pinpoint special learning needs;

(3) to design adequate individualized activities ta -rgeted at improving each student's achievement.

In a diagnostic map (see Table 3) students are arranged in a list according to the points scored, in order to better discern their special learning needs and provide support to students in the learning process. Color marking in a diagnostic map highlights groups of learners and individual learners and enables applying adequate individual measures and teaching methods. The map also highlights those parameters where lack of knowledge, skills and abilities is characteristic of the whole class.

Table 3 reflects the achievement of students in Grade 6b of school X concerning the basic skills in problems 6 and 7 and the solution of problem 8 (a fragment of the electronic table that summarizes diagnostic test results).

Table 3 Fragment of the electronic table that summarizes diagnostic test results in Grade 6b of school X.

	Problem 6					**Problem 7**			Total	**Problem 8**	**Total**	**Total**
	6a	**6b**	**6c**	**6d**	**6e**	**7a**	**7b**	**7c**		**8**		
	1	**1**	**1**	**2**	**2**	**2**	**2**	**2**	36	**5**	**41**	
Performance of tasks (%)	94	94	82	56	53	35	29	44		22		
Student 1	1	1	1	2	2	2	2	2	94%	5	39	95.12%
Student 2	1	1	1	1	2	2	2	2	86%	5	36	87.80%
Student 3	1	1	1	2	2	2	0	0	81%	5	34	82.93%
Student 4	1	1	1	2	0	2	2	2	97%	0	33	80.49%
Student 5	1	1	1	2	2	2	2	2	89%	0	32	78.05%
Student 6	1	1	1	2	2	2	2	2	89%	0	32	78.05%
Student 7	1	1	1	2	2	0	0	0	78%	4	32	78.05%
Student 8	1	1	1	2	1	0	0	0	72%	0	26	63.41%
Student 9	1	0	1	1	2	0	0	0	69%	0	25	60.98%
Student 10	1	1	0	1	2	0	0	1	61%	0	22	53.66%
Student 11	1	1	1	0	0	0	0	0	56%	0	20	48.78%
Student 12	1	1	1	0	0	0	0	0	50%	0	18	43.90%
Student 13	1	1	1	2	0	0	0	2	42%	0	15	36.59%
Student 14	1	1	1	0	0	0	0	2	36%	0	13	31.71%
Student 15	1	1	0	0	1	0	0	0	33%	0	12	29.27%
Student 16	1	1	1	0	0	0	0	0	28%	0	10	24.39%
Student 17	0	1	0	0	0	0	0	0	14%	0	5	12.20%
To compare with rate of solutions to the proposed problems across the country:												
Solution of the problems in Latvia (%)	**96**	**95**	92	53	64	69	56	71		37		

The diagnostic map used to analyze achievement and failings of students in Grade 6b of school X can be seen online [36], a fragment of the diagnostic map is depicted in Table 3. The diagnostic test reveals the percentage and value of problem solution (see last two columns of Table 3). Since problem 8 is creative, a new column is introduced in the table that expresses the total percentage of problem solution for basic problems (36 points). Analysis of data in both total percentage columns and the overall indicators of problem solution to the diagnostic map allows for distinguishing three groups of students depending on their basic skills and ability to solve creative problems:

(1) Students 1 to 6 have learnt basic skills at an optimal level; therefore, they are advised to perform correction of errors independently. In mutual cooperation, these learners are advised to solve higher complexity problems while the rest of the class is engaged with revision and consolidation of basic skills;

(2) Students 7 to 9 have learnt basic skills at a sufficient level, but learners 10 to 12—at a critical level. Calculation skills and abilities in this group are basically learnt, yet students lack the skill of using knowledge in new situations (problem 3b), the skill of marking a simple fraction on the grid (problem 4), the skill of measuring the length of the sides of a triangle and calculating its perimeter (problem 5); also, they have insufficient skills to solve problem 7. Problems 3b, 7 and 8 have caused difficulties for the entire class, so

they are to be solved together in class. To consolidate basic skills in this group, the teacher should use assistants during the lesson—tactful and sensitive students with higher learning achievement;

(3) Students 13 to 17 have failed to learn skills of calculation, which is also manifested in further problem solution. These students can read a column diagram very well (problems 6a, 6b, 6c that is Grade 3a level) but they cannot use the quantities arrived at through calculation in connection with the concept "How many times more?" (problem 6d), neither do they know or are able to calculate the mean value of numbers (problem 6e). These students cannot solve a practical problem (problem 7). They have managed less than 50% of the work amount and at the same time less than 50% of problems that involve basic skills (see the column of Table 3 for basic skills).

Conclusion about group 3: there are 5 students in the class with low academic achievement (29% of the total number of students). With these students, individual work is needed during lessons (first of all, revising operations with rational numbers) as well as consultations.

Those who analyze the diagnostic test lack information about students 15, 16, and 17. Information concerning these students, their attitude towards academic work and class attendance notwithstanding, must be discussed at the "small pedagogical meeting" of school involving the teacher of grades 1-3, the teacher of mathematics, the teacher of natural science and the language teacher grades 4-6, representatives of school administration, social educator, psychologist, and the grade teacher. The teacher of mathematics may often become a scapegoat, even though the causes of the low achievement in the subject may be much more complex.

100% inability to solve the test part of the diagnostic test along with inability to solve other problems can probably be taken as a signal that student 17 needs special assistance.

The basic skills included in problems 6a, 6b, and 6c for Grade 6b of school X have been acquired at an optimal level, while reading a diagram in connection with the use of the concept "How many times more?" in calculations included in problem 6d is acquired at a critical level (53.32%).

Students make the following errors:

(1) In understanding the concept "How many times more?", they do not search for the ratio of the larger number to smaller (larger number divided by the smaller).

(2) They read numbers correctly but write an incorrect ratio.

(3) Instead of the ratio larger to smaller they write a difference between larger and smaller or their multiplication.

(4) They do incorrect division of two natural numbers where quotient is a decimal fraction or a simple fraction.

(5) They misinterpret capacity division, not indicating that the quotient expresses *how many times one quantity is included in another*.

They write the textual answer incorrectly, though it was not even required of them.

Another widespread error from the primary school that is not marked by teachers of mathematics in students' papers:

Doing actions with numbers produces a number! Adding numbers 5 and 3 does not yield Euro. Hence, the name, word, abbreviation of the quantity characterized by the number must be put in brackets. This suggests that such imprecision of writing must be added to the evaluation criteria.

Diagnostic maps can help determine **the dynamics of results.** For instance, having compared the diagnostic maps [14] of the pilot research Grade 6a test 1 (Positive numbers and related actions) to test 2 (Using comparison concepts to solve practical problems), we discerned positive dynamics:

"Pilot research test 2 (comparison and calculation skills) yields positive dynamics as compared to test 2 (calculation skills). The mean coefficient of problem solution has increased from 0.59 to 0.80. The number of students who achieve less than half of standard requirements has dropped from 5 to 1”.

4.3. Methodological Recommendations

To provide support for teachers, methodological recommendations were proposed. Recommendations were provided for teachers before national diagnostic tests in grades 3 and 6; they were published on NECC webpage [37 and 14]. Subsequent to completion of diagnostic tests, analysis of test results in grades 3 and 6 was produced and recommendations for eradication of errors published on NECC webpage [32 and 38]. Methodological recommendations provide a detailed analysis of errors and offers methodological solutions for their eradication.

One of the reasons for errors in the use of the concept “How many times more?” is that students lack a visual preconception, they have never checked how the ratio of two quantities can be measured practically by means of strings, strips, or wires. Another typical situation is wrong statements such as “by 3 times more”. The teacher must be consequent with demanding learners to use statements “more by 3” or “3 times more” depending on respective problem.

more by 3

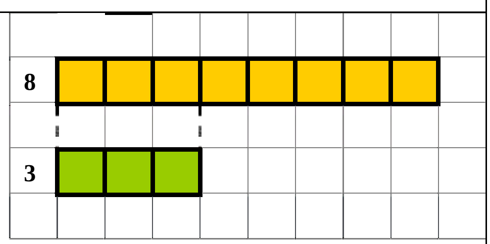
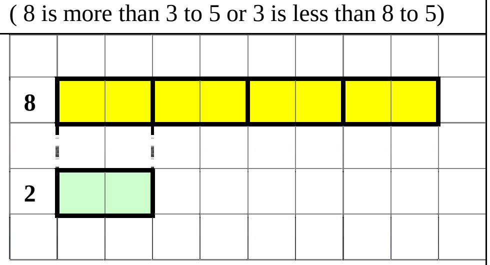
quantity + 3

3 times more (3 such quantities)

quantity + quantity + quantity

Samples of such problems are analyzed in preparation for diagnostic test in both grade 3 and 6.

Table 4 Instruction card sample for learning concepts.

Concept	What is meant by it	Solution expression
More by 3	The same quantity and 3 more	*** + 3**
Less by 3	Lacking 3 for the same quantity	*** – 3**
3 times more	3 such quantities	*** · 3**
3 times less	One third of the quantity	*** : 3**
How much more? How much less?	Difference between the greatest and the least (the least subtracted from the greatest)	 **8 – 3 = 5** (8 is more than 3 to 5 or 3 is less than 8 to 5)
How many times more? How many times less?	Ratio: the greatest vs. the least (the greatest divided by the least)	 **8 : 2 = 4** (so many times is 8 more than 2 or so many times is 2 less than 8)

*** Number according to problem conditions.**

Methodological recommendations for teachers are arranged in thematic groups. They are formulated on the grounds of years of practical experience and stem from analyses of student errors in diagnostic tests. Teachers were suggested both explanations on the causes of errors and indications on how to teach, how to make instruction cards (see Table 4).

Thus, for instance, methodological recommendations for revision of correspondence and comparison concepts and their usage to prepare grade 3 and grade 6 students for diagnostic tests were offered in Table 4, which can be used to consolidate what students learned before.

Methodological suggestions derived from analysis of national diagnostic tests for teachers in grade 6 offer ways of drawing up basic and higher complexity problems by using topical information in papers and magazines. See, for instance, a problem based in real life situations from the group of skills interconnection:

Basic problem: In August 2014, 2000 cyclists were surveyed. Their replies to the question, "Do you wear a reflecting vest in the dark time of the day?" are given in the sector diagram "Reflecting vests for cyclists in the dark time of the day" (see Figure 3).

Higher complexity problem: planned for students with higher academic achievement, as the value of per cent is indicated by decimal fractions and questions 4.2., 4.3. require a deeper analysis of the text.

In August 2014, "BTA Insurance Company" produced an online survey with 2000 respondents. Replies to the question are summarized in Table 5.

4.1. How many respondents do technical maintenance of their bicycle once a year at the beginning of the cycling season?

4.2. How many per cent of respondents say that maintenance of the bicycle is performed less than "At least twice a year"?

4.3. How many times do the respondents perform technical maintenance of their bicycle once a year as compared to those who do it at least twice a year?

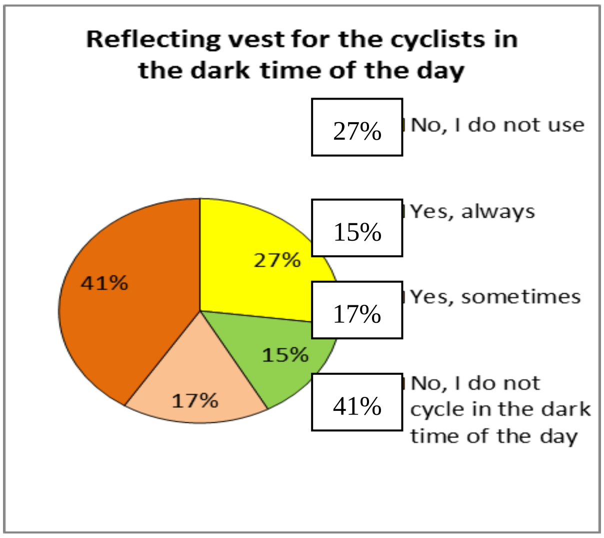
The contribution of purposeful work at school by using national diagnostic tests will be measured only when the present students of grade 3 in 3 years' time are in grade 6, and in 3 more years finish grade 9.

"Many cycles of international research (IEA TIMSS, IEA PIRLS) have revealed the high quality of primary school education (grades 3, 4) and its growth in international comparison. OECD PISA research demonstrates average education quality at the final stage of elementary school in Latvia. Therefore, it is vital to locate and try to eradicate the causes of the drop in quality of education in grades 5-9 in Latvia's schools, as compared to the group of grades 1-4" [39, p. 156].

Study of textbooks and methodological aids for grades 1 to 9 by different authors reveals that their interpretation of the requirements of mathematics standard differs as they place a stronger or weaker emphasis on individual issues and their degree of complexity. Teachers also have their own understanding about the degree of complexity of the problems that must be reached by the end of grades 3 and 6. Consequently, students are often overloaded or fail to meet the standard requirements. Teachers should focus more on working with concepts, because their misunderstanding is a frequent cause of errors. It is important to develop a clear understanding of concepts. One must pay more attention to improving the competence of problem-solving by means of non-standard tasks.

Samples of academic program curricula are at different stages of elaboration:

"Learning objectives must be determined so that it would be possible to diagnose the degree of their attainment. Both teachers and authors of national diagnostic tests must be accurate about what knowledge, skills, and competences are to be reached as a learning outcome, what methodological means are to be used to measure and evaluate the degree of attainment of the planned outcomes..." [40, p. 142]. Teachers need samples of various problems for different levels of acquisition.



3.1. How many per cent of survey participants always use reflecting vests?

3.2. How many survey participants always use reflecting vests?

3.3. The number of which participants of the survey is larger—those who always use reflecting vests or those who sometimes use them? By how many per cent more? By how many participants more?

3.4. How many per cent of the survey participants cycle in the dark time of the day? Justify your answer.

Fig. 3 Diagram for basic problem.

Table 5 Frequency of technical maintenance of a bicycle.

Replies	Replies of the surveyed in %
Less than once a year	16.9%
Once a year at the beginning of the cycling season	46.7%
At least twice a year	20%
More than twice a year	16.4%

5. Conclusion

(1) Acquisition of the notions of comparison during mathematics lessons in elementary school is closely related to students' mental calculation skills as well as to their experience of solving problem situations. In order to reduce the number of students with low achievement, errors should be analyzed more carefully and steps should be taken to develop students' mathematical reasoning.

(2) If learning mathematics is oriented at particular learning outcomes, such methodology should be preferred, which facilitates active learning, critical thinking and students' ability to use the acquired knowledge and skills in problem-solving and everyday life situations.

(3) Diagnostic tests for grades 3 and 6 are organized before the revision work in each stage of learning. They should cover key issues in the curriculum to assess the dynamics of academic achievement and give an opportunity before the end of the year to organize support events that promote mastery of basic skills at an optimal level.

(4) In sample programs of academic subjects and other publications, samples of basic and non-standard problems need to be included, which also ought to reflect the international requirements for students' mathematical competence.

(5) Training seminars ought to be organized for teachers to help them acquire the competence of designing and analyzing test papers by choosing appropriate problems to be solved. The skills necessary to solve these problems may be classified as follows: reproduction, interconnection, mathematical thinking and generalization. This would contribute to the theory and practice of teaching mathematics. The problems must be based on real life situations, so that learning mathematics is related to students' experience.

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