Journal of Mathematical Sciences 1 (2014) 46-52 **Betty Jones & Sisters Publishing**

Mathematical Modeling of Ionic Flows in Troposphere

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Abstract: This paper deals with electrogasdynamical processes in the lower layers of atmosphere. We propose a mathematical model of the ionic flow formed by the ground-based emitter system and study its solutions. The investigation consists in numerical solving of the Navier-Stokes system for the compressible flow coupled with the Nernst-Planck-Poisson equations with respect to the ion concentration and the electric potential. We analyze the final distribution of ions in the gap between the ground surface and the lower cloud boundary. It is shown that considerable amount of ions can reach the height of 2 km only with the ascending air flow. Some temperature effects are described.

Key words: Artificial ionic flow, atmosphere, mathematical model.

Nomenclature

t	Time
z	Height
w	Flow velocity
p	Pressure
e	Specific energy
c	Ion concentration
H	Cloud elevation
R	Universal gas constant
F	Faraday constant
N_A	Avogadro number
g	Gravitational constant
D	Diffusion coefficient

Greek Letters

ρ	Flow density
φ	Electric potential
γ	Adiabate parameter
α	Boundary parameter
ε	Dielectric permittivity
ε_0	Dielectric constant

1. Introduction

Despite the general abundance of water in the world, only a small fraction of it is suitable for human consumption. Nearly 97% of the world water resources consist of the salty sea water. The remaining 3% of water are fresh, but 2.1% of them are frozen and the majority of the remaining 0.9% is underground. The demand for fresh water strictly increases with the population growth. Both the agriculture and the industry need fresh water as well and thus tend to pump it out worldwide [1].

At the same time in the atmosphere there is six times as much water than there is in all the rivers and lakes of the Earth. Therefore the controlled rainfall can solve the problem of the usable fresh water supply. The existing technologies of rain control are based on cloud seeding with silver iodide. This technology works mainly with cold clouds, when the temperature at least in some parts of the cloud is below 0°C. Typically it increases the rainfall by 5 to 14% [2].

A much more efficient approach to the artificial rain problem is based on the formation of the charged particles' flow [3]. Ionized particles move up from the surface and attract water vapor from the circumfluent air. Thus the clouds arise. Under proper conditions the flow movement takes the form of the shock wave propagation. In this case the acoustic waves are generated; they also rapidly enhance the rate of coagulation, and in [4] it is reported that the acoustic

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wave produced by the experimental installation in Armenia reached the ionosphere. There is a controversial evidence of the successful usage of the ionic flow generators in the recent years; in particular of rainfall stimulation in the United Arab Emirates in 2010. The WMO report [5] even doubts the ionization methods of weather modification.

The concept of the rainfall evocation by the charged particles' flow goes back to the IOLA (Ionization of Local Area) technology, proposed in the USSR in 1980-ies. In spite of long and diverse investigations, a lot of important aspects still remain unclear [5]. The present paper deals with one of the particular problems: what initial flow velocity is necessary for maintaining a sufficient concentration of negative ions at the lower cloud edge?

For solving this problem we apply the continuous media approximation: the air flow is described with its density, velocity and pressure while the ions are represented only by concentration because these charged particles are involved into convective movement together with the neutral air molecules. The electric field is expressed through its potential: $\varphi = -\nabla\vec{E}$. This approach leads us to the Navier-Stokes system of equations for the description of the electrified compressible viscous air flow [6] which is coupled with the Nernst-Planck-Poisson electrodiffusion model [7].

2. The Model

We regard the ion flow generator as a ground-based device that produces a vertical air flow with density ρ_0, pressure p_0 and velocity w_0 at the surface level. It also enriches the air with negative oxygen ions up to the concentration c_0. These ions are considered to be an admixture to the flow. But besides the general convective movement the ions are also involved in the diffusive and migrative types of transfer that arise due to the gradients of concentration and electric potential.

We construct the model in Cartesian coordinates with the z-axis pointing up along the normal to the Earth surface, and we neglect the dependence on the tangential variables x and y. To describe the vertical air current we use the Navier-Stokes system with respect to the density, velocity and specific energy of the compressible non-viscous flow; and we use the Nernst-Planck-Poisson system for the description of the convection, diffusion and migration of ions and the evolution of the electric field between the ground and the clouds:

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial z}(\rho w)=0; \tag{1}$$

$$\frac{\partial}{\partial t}(\rho w)+\frac{\partial}{\partial z}(\rho w^2+p)=-g\rho; \tag{2}$$

$$\frac{\partial}{\partial t}(\rho e)+\frac{\partial}{\partial z}\big(w(\rho e+p)\big)=-g\rho w; \tag{3}$$

$$p=(e-\frac{1}{2}w^2)\cdot(\gamma-1)\rho; \tag{4}$$

$$\frac{\partial c}{\partial t}=D\frac{\partial}{\partial z}\left(\frac{\partial c}{\partial z}-\frac{F}{RT}\cdot c\cdot\frac{\partial\varphi}{\partial z}\right)-\frac{\partial}{\partial z}(cw); \tag{5}$$

$$\frac{\partial^2\varphi}{\partial z^2}=\frac{F}{\varepsilon\varepsilon_0 N_A}(c). \tag{6}$$

Our model also includes the following boundary conditions:

$$z=0:\quad \rho=\rho_0,\, w=w_0,\quad \frac{\partial e}{\partial z}=0,\, \frac{\partial c}{\partial z}=0,\quad \varphi=\varphi_0; \tag{7}$$

$$z=H:\quad \frac{\partial\rho}{\partial z}=\alpha\rho w,\, \frac{\partial w}{\partial z}=0,\quad \frac{\partial e}{\partial z}=0,\, \frac{\partial c}{\partial z}=0,\quad \varphi=\varphi_H. \tag{8}$$

The conditions (7) correspond to the experimental installation described in [4, 8]: The flow density and velocity at the ground level are fixed as well as the electric field potential; the ion concentration and the flow temperature (and thus the flow specific energy) are supposed to be constant inside the ground-based device. As far as the upper boundary conditions (8) are concerned, we suppose that the cloud keeps the constant electric potential (which is true for the regarded periods of time). We also suppose that the ion concentration, the velocity and the temperature of the flow stop changing at the height H, because the whole process becomes stable there: the cloud is much greater than the flow and thus quickly adsorbs it.

We approximate the flow temperature in two ways: a) without gradient: $T = T_0 = \text{const}$, and b) with gradient: $T = T_0 - 0.0075 \cdot z$ [6]. In Section 3 we compare the results of these approximations. The elevation H is taken equal to 2,000 m, which is typical for the cloud base [9].

We suppose that the gas dynamical characteristics at the ground level are fully determined. For this reason the flow density ρ and velocity w are set equal to ρ_0 and w_0 at $z = 0$, and the value of the specific energy e is obtained from (4) with $\rho = \rho_0$, $w = w_0$ and $p = p_0$. For the specific energy e and ion concentration c we use the zero gradient boundary conditions. The value of the electric field potential φ at $z = 0$ is assumed to be $\varphi_0 = 0$, whereas its value at $z = H$ is set equal to $\varphi_H = 1.0E + 05V$.

The initial distributions of ρ, w, p and c for $t = 0$ have the stepwise form:

$$\rho(0,z) = \begin{cases} \rho_0 = 4.50 \cdot 10^0 & for\ 0 \le z \le L, \\ \rho_1 = 2.25 \cdot 10^0 & for\ L < z \le H; \end{cases}$$
$$p(0,z) = \begin{cases} p_0 = 2.02 \cdot 10^3 & for\ 0 \le z \le L, \\ p_1 = 1.01 \cdot 10^3 & for\ L < z \le H; \end{cases} \quad (9)$$
$$w(0,z) = \begin{cases} w_0 = 1.00 \cdot 10^0 & for\ 0 \le z \le L, \\ w_1 = 0.00 \cdot 10^0 & for\ L < z \le H; \end{cases}$$
$$c(0,z) = \begin{cases} c_0 = 1.00 \cdot 10^0 & for\ 0 \le z \le L, \\ c_1 = 0.00 \cdot 10^0 & for\ L < z \le H; \end{cases}$$

where $L = 0.01 \cdot H$.

We use the MacCormack finite difference scheme for approximation of the equations (1)-(4) on the discrete mesh with 10^4 nodes in z and with $\Delta t = \Delta z$. On each time layer the equations are solved with respect to ρ, w and e, and the obtained value of w is substituted into the implicit symmetric scheme that approximates (5)-(6). Then the obtained nonlinear equations with respect to c and φ are solved iteratively. This procedure yields the sought solution for the new time layer. The computational process proves to be stable and has the second order of accuracy with respect both to t and z.

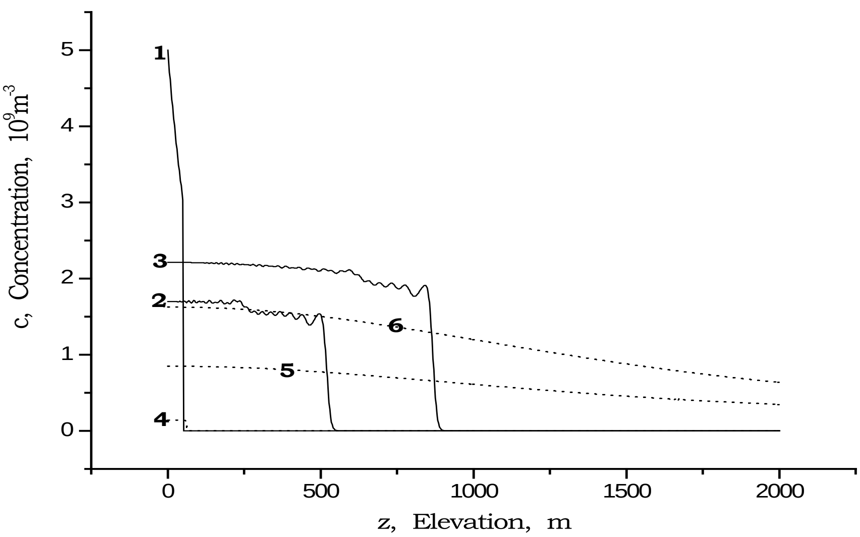

Fig. 1 Ion concentration evolution in time. Curves 1, 2 and 3: time = 120 seconds; surface velocity = 0, 5 and 10 m/sec. Curves 4, 5 and 6: steady state distribution; surface velocity = 0, 5 and 10m/sec.

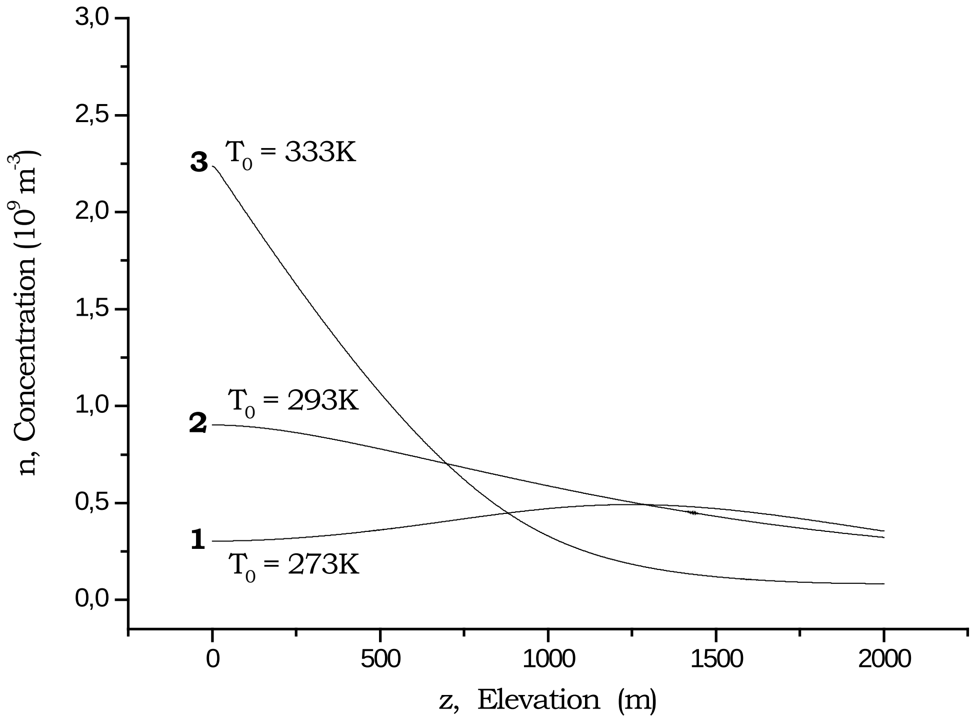

Fig. 2 Steady state ion concentration vs. height. Surface velocity: 5 m/sec. Temperature gradient: $T = T_0$-0.0075·z.

3. Results and Discussion

The equations (1)-(4) with the initial conditions as above describe the propagation of the shock wave in a vertical direction. But in this paper we mostly study the behavior of the admixture that consists of the oxygen ions. These negatively charged particles are involved in the convective movement of the air flow and are simultaneously influenced by the electric and gravity fields. Their transport is described by equations (5)-(6), coupled with (1)-(4). We study the evolution of the ion concentration in time and its dependence of height for different flow temperature distributions and initial flow velocities.

First of all we demonstrate the existence of the critical surface velocity, which is greater that zero, but less than 5 meters per second for the considered parameter values. If the surface velocity of the flow is lower than the critical one, then initial ion concentration step, formed in the interval $0 \le z \le L$ at $t = 0$, never reaches the upper boundary $z = H$. Instead the admixture slowly drifts through the lower boundary due to the migration process. And vice versa, if the surface velocity is greater than critical, then the traveling step is being formed; it moves upward and carries the admixture with it. For the velocities greater than the critical one, the difference in the final density distribution is quantitative, but not qualitative. This is demonstrated in Fig.1, which contains the results obtained for the constant temperature T = 293K. The curves 1, 2, 3 (the surface velocity equals 0, 5 and 10 meters per second respectively) relate to the moment t = 120 seconds. The concentration step moving upwards may be observed only for the two latter cases. The curves 4, 5 and 6 demonstrate the steady state distribution for the same initial velocities. It is easily seen that for both positive velocities the charged admixture becomes more or less evenly distributed along the vertical axis and a sufficient amount of ions reaches the height of 2000 meters, while with zero velocity all the ions leak through the bottom and become lost for cloud transformation.

Another important factor is the temperature of the flow. It is well known that up to the height of 10 kilometers the air temperature T = T(z) obeys the empiric law: $T = T_0 - 0.0075 \cdot z$, where T_0 is the temperature at the surface and z is the distance to this surface. This law is called the temperature gradient. We compare the influence of both terms, the constant T_0 and the variable $0.0075 \cdot z$, on the steady state distribution of the ion concentration. In Fig. 2 we show the results of computations with T = T(z) for T_0 = 273, 293 and 333 K (the curves 1, 2 and 3 respectively). Let us compare the curve 2 in Fig.2 and the curve 5 in Fig.1. They demonstrate the final distribution of ion concentration for the same surface temperature: 293K. It is easily seen that these curves practically coincide for all values of z, and thus the temperature gradient is unsubstantial. At the same time the variation of T_0 causes qualitative difference in the resulting distributions: the curve 3 (T_0 = 333K) decreases approximately according to quadratic law, while the curve 2 (T_0 = 293K) decreases linearly and curve 1 (T_0 = 273K) is not monotonous at all. Therefore the surface temperature is much more important for the ion concentration than the temperature gradient, which may be even ignored in the first approximation.

Similar results are obtained for the steady state flow velocity. The influence of the temperature gradient proves to be insignificant while the temperature and flow velocity at the surface are really important. The curves 1, 2 and 3 in Fig. 3 demonstrate the final distribution of the flow velocity for T = 293K, obtained without temperature gradient with the velocities of 0, 5 and 10 meters per second at the surface. At the same time the curves 1, 2 and 3 in Fig. 4 show the same steady state velocity distributions, obtained with the temperature gradient with T_0 = 273, 293 and 333 K and with the velocity equal to 5 meters per second at the surface. It is easily seen that both curves number 2 - in Fig.3 and Fig.4 - practically coincide. These curves relate to the same surface temperature and velocity, thus the temperature gradient doesn't influence the steady state distribution of flow velocity as well as of ion concentration.

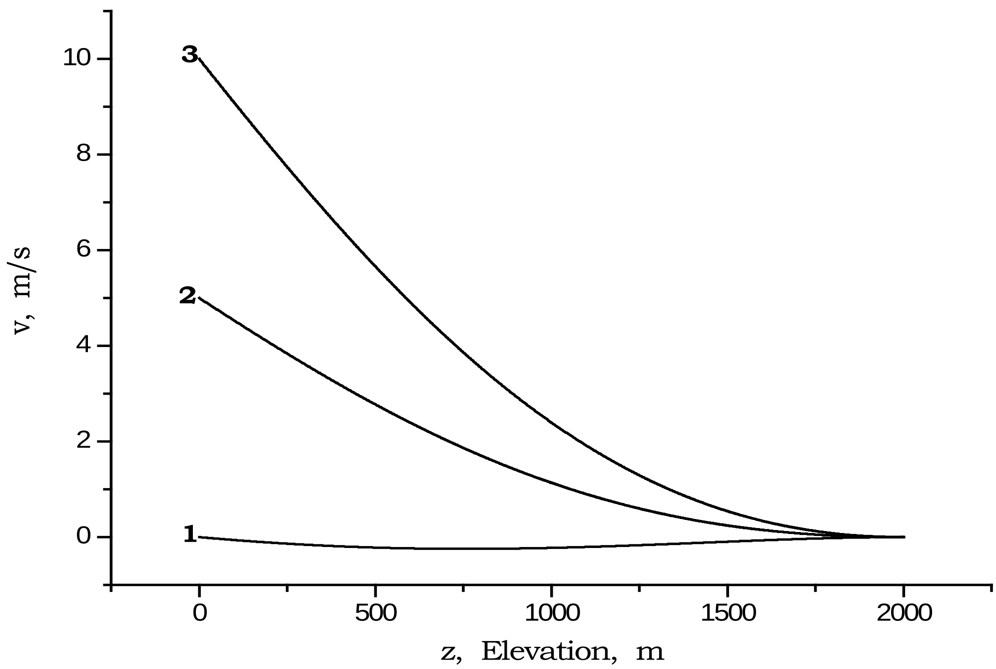

Fig. 3 Steady state flow velocities vs height. Curves 1,2 and 3: initial velocity = 0, 5 and 10 m/sec; T = 293K, no gradient.

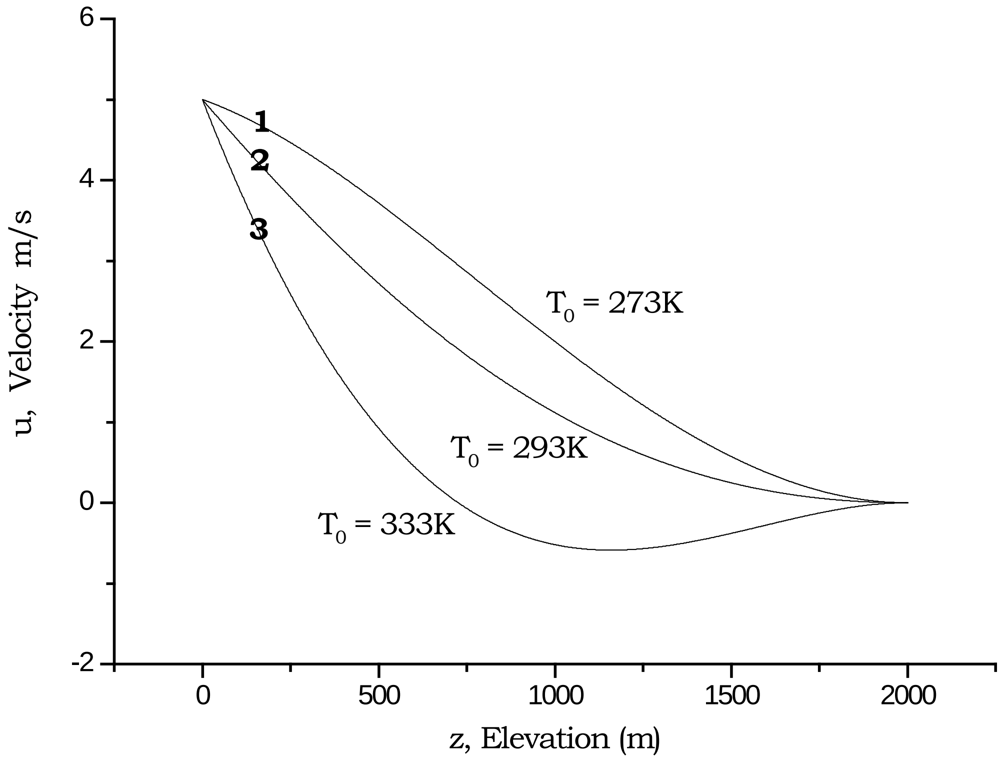

Fig. 4 Steady state flow velocity vs. height. Surface velocity: 5 m/sec. Temperature gradient: $T = T_0$-0.0075·z..

Finally we would like to point out that the convection, albeit an important factor of the charged admixture transport, is not the only significant one. Comparing Fig.2 and Fig.4, we see that for T_0 = 333K there exists an interval of heights from 750 m to 2000 m, in which the air flow velocity is slightly negative. Never the less, the ions successfully overcome this gap due to the ion diffusion and the migration in the electric field. After the passage of the shock wave, which is accompanied by the concentration step movement (as it is shown in Fig.1), the whole system relaxes to the stable steady state distribution of all its characteristics. But this state is spatially non-uniform (see, for example, the distribution of ion concentration in Fig.2). Ion transport due to the gradients of the concentration and of the electric field potential compensates the convective transfer, and the right hand side of (5) turns into zero for all values of z.

4. Conclusions

The solutions to our mathematical model of the oxygen ionic flow produced by ground emitters clearly demonstrate that a considerable amount of ions can reach the height of two kilometers, given that the initial velocity of the flow is about 5 meters per second. The absolute temperature of the flow proves to be another crucial transport parameter while the temperature gradient seems to be not so important. Starting from the height of approximately 750 meters the role of the electric field becomes determinant for the ion transportation.

5. Future Work

More accurate results may be obtained in such aspects as the critical value of the flow velocity at the surface and the influence of the absolute temperature and the electric potential on the initial wave propagation. But it should be done in another modeling framework: in cylindrical coordinates, taking into account the sidewind and other possible ion leakages, and using a more accurate finite difference approximation [10] for removing the artificial density oscillations. Also some other boundary conditions should be tried for the proper description of the flow interaction with the cloud. We plan to introduce these improvements in our successive investigations.

Acknowledgments

The authors are very grateful to Dr. R.N.Kuzmin from the Department of Physics, Lomonosov Moscow State University, for many consultations and fruitful discussions, which have improved this paper in many aspects.

References

[1] UN World Water Development Report 3: Water in a Changing World. WWAP. 2009.

[2] B. Vonnegut, H. Chessin, Ice nucleation by coprecipitated silver iodide and silver bromide, Science 174 (1971) 945-946.

[3] H. Jung, D.B. Kittelson, Characterization of aerosol surface instruments in transition regime, Aerosol Science and Technology 39 (2005) 902-911.

[4] G.A. Galechyan, The analogy between the processes induced in a gas discharge by a gas flow and by acoustic waves, Laser Physics 4 (1994) 23-37.

[5] World Meteorological Organization. WMO Documents on Weather Modification. Report of the Expert Team on Weather Modification Research, Abu Dhabi, 22-24 March 2010; final revision-27 April 2010.

[6] Hsiao Ling, Li Hailiang, Compressible Navier-Stokes-Poisson Equations, Acta Mathematica Scientica 30B (2010) 1937-1948.

[7] I. Rubenstein. Electrodiffusion of ions, SIAM Studies in Applied Mathematics 11 (1990).

[8] G.A. Galechyan, A.A. Vardanyan, Flows of negative ions for stimulation of the precipitation, International Scientific Journal for Alternative Energy and Ecology 8 (2012) 86-137.

[9] Yu.S. Sedunov, Atmosphere Handbook, Hydrometeoizdat, Leningrad, USSR, 1991.

[10] V.A. Bityurin, A.N. Bocharov, N.A. Popov, Numerical simulation of an electric discharge in supersonic flow, Fluid Dynamics 43 (2008) 642-653.