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Analytic Similarity Solution of a Spherical-Cylindrical Radiation-Driven Shock propagating in an Inhomogeneous Medium

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Abstract: The flow behind the radiation-driven spherical and cylindrical shocks and detonation wave, propagating into inhomogeneous medium whose ambient density law is $\rho_0 = Ar_0^{-\alpha}$ has been studied. An analytic similarity solution of this problem has been obtained by employing Chernyii's technique, in which the flow variables are expanded in series of powers of ε , the density ratio across the strong shock.

Key words: Self-similar, detonation, similarity solution, Chernyii's Technique, Chapman-Jouguet condition.

Subject Classification:

1. AMS (American Maths Society)	76L05 (Shock Waves and Blast Wave)
2. PACS (Phy. & Astron. Classification Scheme) AIP (American Institute of Physics)	47.40.-x (Compressible Flow, Shock and Detonation Phenomena)

1. Introduction

There is an increasing interest in the problems of implosion and explosion. The former, besides having numerous applications, is viewed as a necessary step in realizing controlled thermonuclear fusion. Similarity solutions for the self-similar flows of first and second kind have been given by Sedov [1], Sakurai [2], Zel'dovich and Raizer [3], Sachdev [4], and Vishwa karma and Pathak [5]. A similarity solution for a strong spherical detonation wave, propagating with a constant Chapman-Jouguet speed, has been obtained by Zel'dovich [6] and Taylor [7]. Wilson and Turcotte [8] have studied the problem of a strong spherical radiation-driven shock, generated at the origin, where they assume that radiation is propagated radially inwards through a homogeneous gas with constant power P and that the medium is optically transparent. The incident radiation is completely absorbed within the shock layer and re-radiation from the shock layer is not considered. The behaviour of the shock is similar to that of detonation [9]. For the jump condition they have used the solution obtained by Ramsden and Savic [10] and Raizer [11] for the steady one dimensional propagation of similar type of shock. Recently, some significant contributions in the topic have been carried out by Hirschler and Gretler [12], Levin and Skopina [13] and Bhagatwalaa and Lee [14].

In this paper we have studied the flow behind the radiation driven spherical and cylindrical shocks,

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propagating into inhomogeneous medium whose ambient law is

$$\rho_0 = A r_0^{-\alpha} \tag{1}$$

where A and α are positive constants and r_0 is the distance of the particle from the center (axis) at the initial instant of time. The numerical solution of this problem was given by Wilson and Turcotte [8], for a uniform ambient medium and for the spherical case only. We have obtained approximate analytic solution of the problem by employing Chernyii's [15] technique, in which the flow variable are expressed as a series in ε, the density ratio across the strong shock.

A spherical (cylindrical) shock may be generated by a number of lasers focused on a common point (axis). The initial wave may be generated by the spontaneous breakdown of the gas or by the use of a small absorbing particle. The speed of the radiation driven shock is a function of time because the flux density of the radiation at the shock decreases with time. The flow behind the shock wave is particle isentropic and the jump conditions are matched accordingly. The similarity solution is valid as long as the shock is strong.

2. Basic Equations and Governing Parameters

For the expansion of strong shock wave the governing parameters are taken as the power of incident radiation P and the ambient density ρ_0. From dimensional consideration, the position of shock wave as a function of time is given by

$$R \sim \left(\frac{P}{\rho_0}\right)^{1/5} t^{3/5} \tag{2.1}$$

where R is the distance of the shock from the center (axis). The velocity of the shock wave U_S is given by

$$U_s \sim \left(\frac{P}{\rho_0}\right)^{1/5} t^{-2/5} \tag{2.2}$$

There is no radiation behind the shock wave as it is assumed that the incident radiation is completely absorbed by the shock layer and the flow is particle isentropic. Re-radiation from the shock is not considered. The basic equations governing the flow behind the expanding shock in terms of the Lagrangian coordinate η and time t are

$$\frac{\partial r}{\partial \eta} = \frac{1}{\rho r^{j-1}} \tag{2.3}$$

$$\frac{\partial^2 r}{\partial t^2} = -r^{j-1}\frac{\partial p}{\partial \eta} \tag{2.4}$$

$$\frac{\partial (p\rho^{-\gamma})}{\partial t} = 0 \tag{2.5}$$

where r is the distance of the particle from the center (axis) of symmetry, ρ and p are the density and pressure behind the shock. It is assumed that the ideal gas approximation is valid with the ratio of specific heats. The Lagrangian coordinate η is defined by

$$d\eta = \rho_0 r_0^{\,j-1} dr_0 = A r_0^{\,j-\alpha-1} dr_0 \tag{2.6}$$

where ρ_0 is the ambient density given by equation (1), r_0 is the value of r at initial instant of time and j = 2, 3 for cylindrical and spherical symmetry. We introduce the similarity variable μ defined by

$$\mu = \eta / \eta_s \tag{2.7}$$

where η_s is the value of η at the stock and is given by

$$\eta_s = \frac{A R^{j-\alpha}}{j-\alpha} \tag{2.8}$$

where R is the radial (axial) distance of the shock.

The Eulerian similarity variable λ and the position of the shock wave are given by

$$\lambda = \left(\frac{\rho_0}{p}\right)^{1/5} \frac{r}{t^{3/5}} \tag{2.9}$$

$$R = \lambda_s \left(\frac{p}{\rho_0}\right)^{1/5} t^{3/5} \tag{2.10}$$

Where λ_s is the value of λ at the shock and *t* is the parameter.

We assume the similarity form of flow variables as

$$\begin{aligned} r &= R\frac{\lambda}{\lambda_s} \\ u &= \dot{R}\bar{u}(\mu) \\ \rho &= \rho_0\bar{\rho}(\mu) \\ p &= \rho_0\dot{R}^2\bar{p}(\mu) \\ T &= \dot{R}^2\bar{T}(\mu) \end{aligned} \tag{2.11}$$

For the boundary conditions we use Rankine-Hugoniot conditions expressed in terms of the parameter λ_s as

$$\bar{u}_1 = \frac{3}{5(\lambda+1)} + \left[\left(\frac{3}{5(\lambda+1)}\right)^2 - \frac{5}{6\pi\lambda_s^5}\left(\frac{\gamma-1}{\gamma+1}\right)\right]^{1/2} \tag{2.12}$$

$$\bar{\rho}_1 = \left[\frac{\gamma}{\gamma+1} - \left\{\left(\frac{1}{\gamma+1}\right)^2 - \frac{125}{54\pi\lambda_s^5}\left(\frac{r-1}{\gamma+1}\right)\right\}^{1/2}\right]^{-1} \tag{2.13}$$

$$\bar{p}_1 = \frac{3}{5}\left[\frac{3}{5(\gamma+1)} + \left\{\left(\frac{3}{5(\gamma+1)}\right)^2 - \frac{5}{6\pi\lambda_s^5}\left(\frac{r-1}{\gamma+1}\right)\right\}^{1/2}\right] \tag{2.14}$$

$$\bar{T}_1 = \lambda\left[\frac{3}{5(\gamma+1)} + \left\{\left(\frac{3}{5(\gamma+1)}\right)^2 - \frac{5}{6\pi\lambda_s^5}\left(\frac{r-1}{\gamma+1}\right)\right\}^{1/2}\right] \tag{2.15}$$

Following Chernyii's [15] technique we expand flow variable in series of power of ε, the density ratio across the strong shock as

$$\lambda = \lambda^{(0)} + \varepsilon\lambda^{(1)} + \varepsilon^2\lambda^{(2)} + \cdots \tag{2.16}$$

$$\bar{u} = \bar{u}^{(0)} + \varepsilon\bar{u}^{(1)} + \varepsilon^2\bar{u}^{(2)} + \cdots \tag{2.17}$$

$$\bar{p} = \bar{p}^{(0)} + \varepsilon\bar{p}^{(1)} + \varepsilon^2\bar{p}^{(2)} + \cdots \tag{2.18}$$

$$\bar{\rho} = \frac{\bar{\rho}^{(0)}}{\varepsilon} + \bar{\rho}^{(1)} + \varepsilon\bar{\rho}^{(2)} + \cdots \tag{2.19}$$

where

$$\varepsilon = \frac{\gamma}{\gamma+1} - \left[\left(\frac{1}{\gamma+1}\right)^2 - \frac{125}{54\pi\lambda_s^5}\left(\frac{r-1}{\gamma+1}\right)\right]^{1/2}$$

Substituting the expansion (2.16) - (2.19) in Rankine Hugoniot (2.12) - (2.14) we obtain the boundary conditions at the shock as

$$\begin{aligned} &\lambda^{(0)} = \lambda_s \\ &\lambda^{(1)} = \lambda^{(2)} = \cdots\cdots\cdots = 0 \\ &\bar{u}^{(0)} = \frac{3}{5(\gamma+1)} + \left\{\left(\frac{3}{5(\gamma+1)}\right)^2 - \frac{5}{6\pi\lambda_s^5}\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^{1/2} \\ &\bar{u}^{(1)} = \bar{u}^{(2)} = \cdots\cdots\cdots = 0 \\ &\bar{\rho}^{(0)} = 1 \\ &\bar{\rho}^{(1)} = \bar{\rho}^{(2)} = \cdots\cdots\cdots = 0 \\ &\bar{p}^{(0)} = \frac{3}{5}\left[\frac{3}{5(\gamma+1)} + \left\{\left(\frac{3}{5(\gamma+1)}\right)^2 - \frac{5}{6\pi\lambda_s^5}\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^{1/2}\right] \\ &\bar{p}^{(1)} = p^{(2)} = \cdots\cdots\cdots = 0 \end{aligned} \tag{2.20}$$

Equation (2.3) and (2.4) may be expressed in terms of particle velocity u, as

$$\frac{\partial u}{\partial \eta} = \frac{-1}{r^{j-1}\rho^2}\left[\frac{\partial \rho}{\partial t} + (j-1)\frac{\rho u}{r}\right] \tag{2.21}$$

$$\frac{\partial u}{\partial t} = -r^{j-1}\frac{\partial p}{\partial \eta} \tag{2.22}$$

We substitute equation (2.11) into equations (2.3), (2.21), (2.22) and (2.5) and by using equations (1), (2.6), (2.7) and (2.8), we obtain the following set of differential equations:

$$\frac{d\lambda}{d\mu} = \frac{\lambda_s{}^{j}}{(j-\alpha)\bar{\rho}\ \lambda^{j-1}} \quad (2.23)$$

$$\frac{d\bar{u}}{d\mu} = \frac{\lambda_s{}^{j-1}}{(j-\alpha)\lambda^{j-1}\bar{\rho}^2}\left[\alpha\bar{\rho} + (j-\alpha)\mu\frac{d\bar{\rho}}{d\mu} - (j-1)\lambda_s\frac{\bar{\rho}\,\bar{u}}{\lambda}\right] \quad (2.24)$$

$$\frac{d\bar{p}}{d\mu} = \frac{\lambda_s{}^{j-1}}{\lambda^{j-1}(j-\alpha)}\left[\left(\frac{\alpha-2}{3}\right)\bar{u} + (\alpha-j)\mu\frac{d\bar{u}}{d\mu}\right] \quad (2.25)$$

$$\left[\frac{3(\gamma-1)\alpha-4}{3(\alpha-j)}\right]\bar{p} + \mu\frac{d\bar{p}}{d\mu} - \frac{\lambda\bar{p}\mu}{\bar{\rho}}\frac{d\bar{\rho}}{d\mu} = 0 \quad (2.26)$$

Substituting expansions (2.16) - (2.19) into equation (2.23) - (2.26) we get the following differential equations:

$$\frac{d\lambda^{(0)}}{d\mu} = 0 \quad (2.27)$$

$$\frac{d\bar{u}^{(0)}}{d\mu} = 0 \quad (2.28)$$

$$\frac{d\bar{p}^{(0)}}{d\mu} = \frac{\lambda_s{}^{j-1}}{(\alpha-j)\lambda^{(0)j-1}}\left[\frac{(\alpha-2)}{3}\bar{u}^{(0)} + (\alpha-j)\mu\frac{d\bar{u}^{(0)}}{d\mu}\right] \quad (2.29)$$

$$\frac{d\lambda^{(1)}}{d\mu} = \frac{\lambda_s{}^{j}}{(j-\alpha)\bar{\rho}^{(0)}\lambda^{(0)j-1}} \quad (2.30)$$

$$\frac{d\bar{u}^{(1)}}{d\mu} = \frac{\lambda_s{}^{j-1}}{(j-\alpha)\lambda^{(0)j-1}\bar{\rho}^{(0)}}\left[\alpha\bar{\rho}^{(0)} + (j-\alpha)\mu\frac{d\bar{\rho}^{(0)}}{d\mu} - (j-1)\lambda_s\bar{\rho}^{(0)2}\frac{\bar{u}^{(0)}}{\bar{\lambda}^{(0)}}\right] \quad (2.31)$$

$$\frac{d\bar{p}^{(1)}}{d\mu} = \frac{\lambda_s{}^{j-1}}{(\alpha-j)\lambda^{(0)j-1}}\left[\begin{array}{c}\frac{(\alpha-2)}{3}\bar{u}^{(1)} + (\alpha-j)\mu\frac{d\bar{u}^{(1)}}{d\mu} - 1 \\ +(j-1)\frac{\lambda^{(1)}}{\lambda^{(0)}}\left\{\begin{array}{c}\frac{(\alpha-2)}{3}\bar{u}^{(0)} \\ +(\alpha-j)\mu\frac{d\bar{u}^{(0)}}{d\mu}\end{array}\right\}\end{array}\right] = 0 \quad (2.32)$$

$$B\bar{p}^{(0)} + \mu\left(\frac{d\bar{p}^{(0)}}{d\mu} - \frac{\gamma\bar{p}^{(0)}}{\bar{\rho}^{(0)}}\frac{d\bar{\rho}^{(0)}}{d\mu}\right) = 0 \quad (2.33)$$

$$B\bar{p}^{(1)} + \mu\left(\frac{d\bar{p}^{(1)}}{d\mu} - \frac{\gamma\bar{p}^{(0)}}{\bar{\rho}^{(0)}}\frac{d\bar{\rho}^{(1)}}{d\mu}\right) - \frac{\gamma\bar{p}^{(1)}}{\bar{\rho}^{(0)}}\frac{d\bar{\rho}^{(0)}}{d\mu} + \frac{\gamma\,\bar{p}^{(0)}\bar{\rho}^{(1)}}{\bar{\rho}^{(0)2}}\frac{d\bar{\rho}^{(0)}}{d\mu} = 0 \quad (2.34)$$

where $B = \dfrac{3(\gamma-1)\alpha-4}{3(\alpha-j)}$.

We integrate equations (2.27) - (2.34) with boundary conditions (2.20) and obtain the solutions as:

$$\lambda^{(0)} = \lambda_s \tag{2.35}$$

$$\bar{u}^{(0)} = A \tag{2.36}$$

$$\bar{p}^{(0)} = C[1 + D(1-\mu)] \tag{2.37}$$

$$\bar{\rho}^{(0)} = \mu^m [1 + D(1-\mu)]^{1/\gamma} \tag{2.38}$$

$$\bar{\lambda}^{(1)} = E\int_1^\mu \frac{\mu^{-m}}{[1+D(1-\mu)]^{1/\gamma}}\,d\mu \tag{2.39}$$

$$\bar{u}^{(1)} = F\int_1^\mu \frac{\mu^{-m}}{[1+D(1-\mu)]^{1/\gamma}}\,d\mu - \frac{\mu^{1-m}}{[1+D(1-\mu)]^{1/\gamma}} + 1 \tag{2.40}$$

$$\bar{p}^{(1)} = G(\mu-1)\int_1^\mu \frac{\mu^{-m}}{[1+D(1-\mu)]^{1/\gamma}}\,d\mu + H\int_1^\mu \frac{\mu^{1-m}}{[1+D(1-\mu)]^{1/\gamma}}\,d\mu - \frac{\mu^{2-m}}{[1+D(1-\mu)]^{1/\gamma}} + L(M-1) + 1 \tag{2.41}$$

$$\bar{\rho}^{(1)} = \frac{\bar{\rho}^{(0)}\bar{p}^{(1)}}{\gamma\bar{p}^{(0)}} \tag{2.42}$$

where $m = \dfrac{B}{\gamma}$.

$$A = \frac{3}{5(\gamma+1)} + \left\{\left(\frac{3}{5(\gamma+1)}\right)^2 - \frac{5}{6\pi\lambda_s^{\,5}}\left(\frac{\gamma-1}{\gamma+1}\right)\right\}^{1/2}$$

$$C = \frac{3}{5}A, \qquad D = \frac{(\alpha-2)A}{3C\,(j-\alpha)}, \qquad E = \frac{\lambda_s}{(j-\alpha)}, \qquad F = 1 + \frac{\alpha-(j-1)A}{(j-\alpha)}$$

$$G = \frac{(\alpha-2)F}{3(\alpha-j)} - \frac{(j-1)(\alpha-2)\,AE}{3(\alpha-j)\lambda_s}$$

$$H = \frac{\alpha-(j-1)A}{(j-\alpha)} - \frac{(\alpha-2)}{3(\alpha-j)}$$

$$L = \frac{(\alpha-2)}{3(\alpha-j)}$$

$$M = 2 + H - G$$

We apply the transformation $Z = 1 + D\,(1-\mu)$ and express solutions (2.39) - (2.42) in terms of incomplete Beta functions as

$$\lambda^{(1)} = A_1\left[B\varsigma(p,q) - B_\beta(p,q)\right] \tag{2.43}$$

$$\bar{u}^{(1)} = B_1\left[B_\zeta(p,q) - B_\beta(p,q)\right] + C_1\xi^{-1/\gamma}[1 + D - \xi]^{1-m} + 1 \quad (2.44)$$

$$\bar{p}^{(1)} = D_1(1-\xi)\left[B_\zeta(p,q) - B_\beta(p,q)\right] + E_1\left[B_\zeta(p,s) - B_\beta(p,s)\right] + F_1\xi^{-1/\gamma}[1 + D - \xi] + G_1(1-\xi) + 1 \quad (2.45)$$

$$\bar{\rho}^{(1)} = \frac{1}{\gamma\, C D^m}\xi^{\frac{1-\gamma}{\gamma}} + [1 + D - \xi]\left[D_1(1-\xi)\{B_\zeta(p,q) - B_\beta(p,q)\} + F_1\xi^{-1/\gamma}(1 + D - \xi) + G\,(1-\xi) + 1\right] \quad (2.46)$$

where $\zeta = \frac{\xi}{1+D}$, $\beta = \frac{1}{1+D}$, $p = 1 - \frac{1}{\gamma}$

q = 1-m, s = 2-m ,

$$A_1 = \frac{-E(1+D)^{p+q-1}}{D^{1-m}}, \quad B_1 = \frac{-F(1+D)^{p+q-1}}{D^{1-m}}, \quad C_1 = \frac{-1}{D^{1-m}},$$

$$D_1 = \frac{-G(1+D)^{p+q-1}}{D^{2-m}}, \quad E_1 = \frac{-M(1+D)^{p+s-1}}{D^{2-m}}, \quad F_1 = \frac{-1}{D^{2-m}}, \quad G_1 = \frac{L}{D}$$

Thus the sets of equation (2.35) - (2.36) and (2.43) - (2.46) give the solution, up to first order in **ε**, in closed form in terms of incomplete Beta function. The dimensionless position of the shock wave is given by $\lambda_s = 1.067$ for $\gamma = 5/3$. When -1 < D < O, we may obtain the solution by putting D = - D', where D' > O.

3. Chapman-Jouguet Condition:

Raizer [11] has shown that a shock wave driven by radiation (absorption wave) has Hugoniot curve similar to that for detonation wave. He has further shown that for a given energy flux there is a minimum possible propagation speed, which corresponds to the Chapman-Jouguet condition. The Chapman-Jouguet condition requires that the shock velocity relative to the heated fluid behind it be identical to the local sound speed, i.e.,

$$a_{1cj} = [\gamma\, \frac{p_{1cj}}{\rho_{1cj}}]^{1/2} \quad (3.1)$$

If the shock wave propagates at the Chapman -Jouguet speed the down-stream flow is sonic in shock-fixed coordinates and this condition gives the value of the dimensionless position of the shock wave as

$$\lambda_{scj} = [\frac{125}{54\pi}(\gamma^2 - 1)]^{1/5} \quad (3.2)$$

This value of λ_s transforms the reduced flow variables (2.12)-(2.15), immediately behind the shock to

$$\bar{u}_{1cj} = \frac{3}{5(\gamma+1)} \quad (3.3)$$

$$\bar{\rho}_{1cj} = \frac{(\gamma+1)}{\gamma} \quad (3.4)$$

$$\bar{p}_{1cj} = \frac{9}{25(\gamma+1)} \quad (3.5)$$

$$\bar{T}_{1cj} = \left[\frac{3\gamma}{5(\gamma+1)}\right]^2 \quad (3.6)$$

where the suffix 1 denotes the condition immediately down-stream of the shock layer.

Substituting expansion (2.16) - (2.19) in equation (3.3) to (3.6), we obtain the boundary conditions at the Chapman-Jouguet wave as

$$\lambda^{(0)} = \lambda_s$$

$$\lambda^{(1)} = \lambda^{(2)} = \cdots\cdots\cdots = 0$$

$$\bar{u}^{(0)} = \frac{3}{5(\gamma+1)}$$

$$\bar{u}^{(1)} = \bar{u}^{(2)} = \cdots\cdots\cdots = 0 \qquad (3.7)$$

$$\bar{p}^{(0)} = \frac{9}{25(\gamma+1)}$$

$$\bar{p}^{(1)} = p^{(2)} = \cdots\cdots\cdots = 0$$

$$\bar{\rho}^{(0)} = 1$$

$$\bar{\rho}^{(1)} = \bar{\rho}^{(2)} = \cdots\cdots\cdots = 0$$

We solve the differential equations (2.27) - (2.34) with boundary conditions (3.7) and making use of the transformation Z = 1 + D (1-μ), we express the solution in terms of incomplete Beta function as

$$\lambda_{Cj}{}^{(0)} = \lambda_s \qquad (3.8)$$

$$\bar{u}_{cj}{}^{(0)} = \bar{A} \qquad (3.9)$$

$$\bar{p}_{cj}{}^{(0)} = \bar{C}[1 + \bar{D}(1-\mu)] \qquad (3.10)$$

$$\bar{\rho}_{cj}{}^{(0)} = \mu^m [1 + \bar{D}(1-\mu)]^{1/\gamma} \qquad (3.11)$$

$$\lambda_{cj}{}^{(1)} = \bar{A}_1\left[B_{\bar{\zeta}}(p,q) - B_{\bar{\beta}}(p,q)\right] \qquad (3.12)$$

$$\bar{u}_{cj}{}^{(1)} = \bar{B}_1\left[B_{\bar{\zeta}}(p,q) - B_{\bar{\beta}}(p,q)\right] + \bar{C}_1 \bar{\xi}^{-1/\gamma}\left[1 + \bar{D} - \bar{\xi}\right] + 1 \qquad (3.13)$$

$$\bar{p}_{cj}{}^{(1)} = \bar{D}_1(1-\bar{\xi})\left[B_{\bar{\zeta}}(p,q) - B_{\bar{\beta}}(p,q)\right] + \bar{E}_1\left[B_{\bar{\zeta}}(p,s) - B_{\bar{\beta}}(p,s)\right] + \bar{F}_1\bar{\xi}^{-1/\gamma}\left[1 + \bar{D} - \bar{\xi}\right] + \bar{G}_1(1-\bar{\xi}) + 1 \qquad (3.14)$$

$$\bar{\rho}_{cj}{}^{(1)} = \frac{1}{\gamma\bar{C}\bar{D}}\bar{\xi}^{\frac{1-\gamma}{\gamma}} + \left[1 + \bar{D} - \bar{\xi}\right] \begin{bmatrix} \bar{D}_1(1-\bar{\xi})\{B_{\bar{\zeta}}(p,q) - B_{\bar{\beta}}(p,q)\} \\ +\bar{E}_1\{B_{\bar{\zeta}}(p,s) - B_{\bar{\beta}}(p,s)\} \\ +\bar{F}_1\bar{\xi}^{-1/\gamma}(1 + \bar{D} - \bar{\xi}) + \bar{G}_1(1-\bar{\xi}) + 1 \end{bmatrix} \qquad (3.15)$$

where $\bar{\zeta} = \frac{\bar{\xi}}{1+\bar{D}}$, $\bar{\beta} = \frac{1}{1+\bar{D}}$

$$\bar{A}_1 = \frac{-\bar{E}(1+\bar{D})^{p+q-1}}{\bar{D}^{\,1-m}}, \quad \bar{B}_1 = \frac{-\bar{F}(1+\bar{D})^{p+q-1}}{\bar{D}^{\,1-m}},$$

$$\bar{C}_1 = \frac{-1}{\bar{D}^{\,1-m}}, \quad \bar{D}_1 = \frac{-\bar{G}\,(1+\bar{D})^{p+q-1}}{\bar{D}^{\,2-m}},$$

$$\bar{E}_1 = \frac{-\bar{M}(1+\bar{D})^{p+s-1}}{\bar{D}^{\,2-m}},$$

$$\bar{F}_1 = \frac{-1}{\bar{D}^{\,2-m}}, \qquad \bar{G}_1 = \frac{\bar{L}}{\bar{D}}$$

4. Results and Discussion

Equations (3.8) - (3.15) give the reduced λ_{cj}, reduced velocity, pressure and density in closed form in terms of incomplete Beta functions for the Chapman-Jouguet shock wave in non-uniform medium. The value of dimensionless position of a Chapman-Jouguet shockwave is $\lambda_s = 1.055$ for spherical case for $\gamma = 5/3$.

We have obtained approximate analytic solutions of spherical and cylindrical radiation driven shocks, in closed form, in terms of incomplete Beta function up to first order term in ε, the density ratio across the strong shock. Equations (2.35) - (2.42) give similarity variable λ, reduced particle velocity, pressure and density behind the shock as a function of μ, the Lagrangian similarity variable. The error in the solution is of the order of ε^2 which is small if γ is o (1). We have also obtained separate solutions for Chapman-Jouguet shock wave.

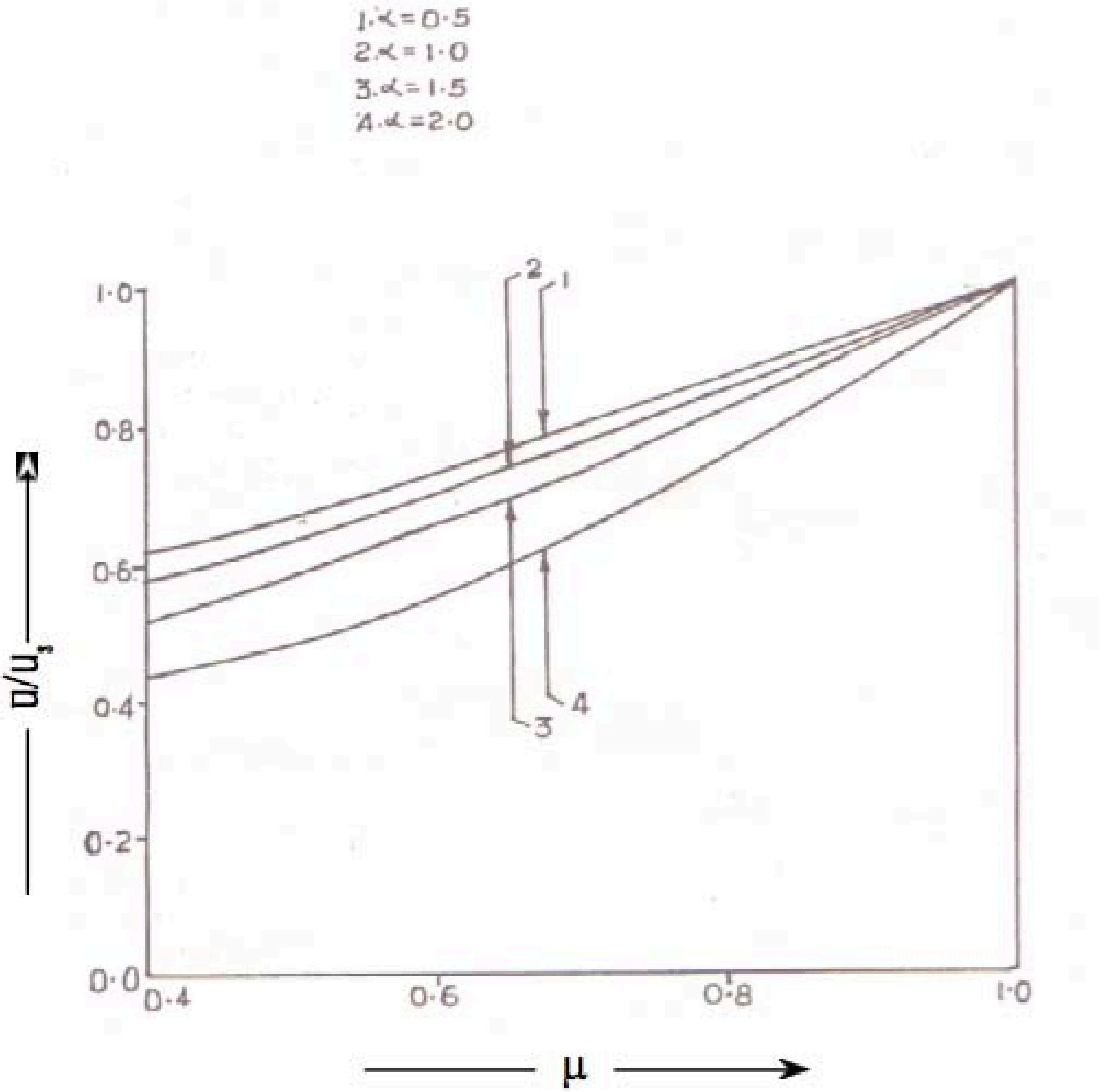

FIG - 1

VELOCITY DISTRIBUTIONS BEHIND THE SPHERICAL SHOCK WAVE AS FUNCTION OF μ FOR $\gamma = 5/3$

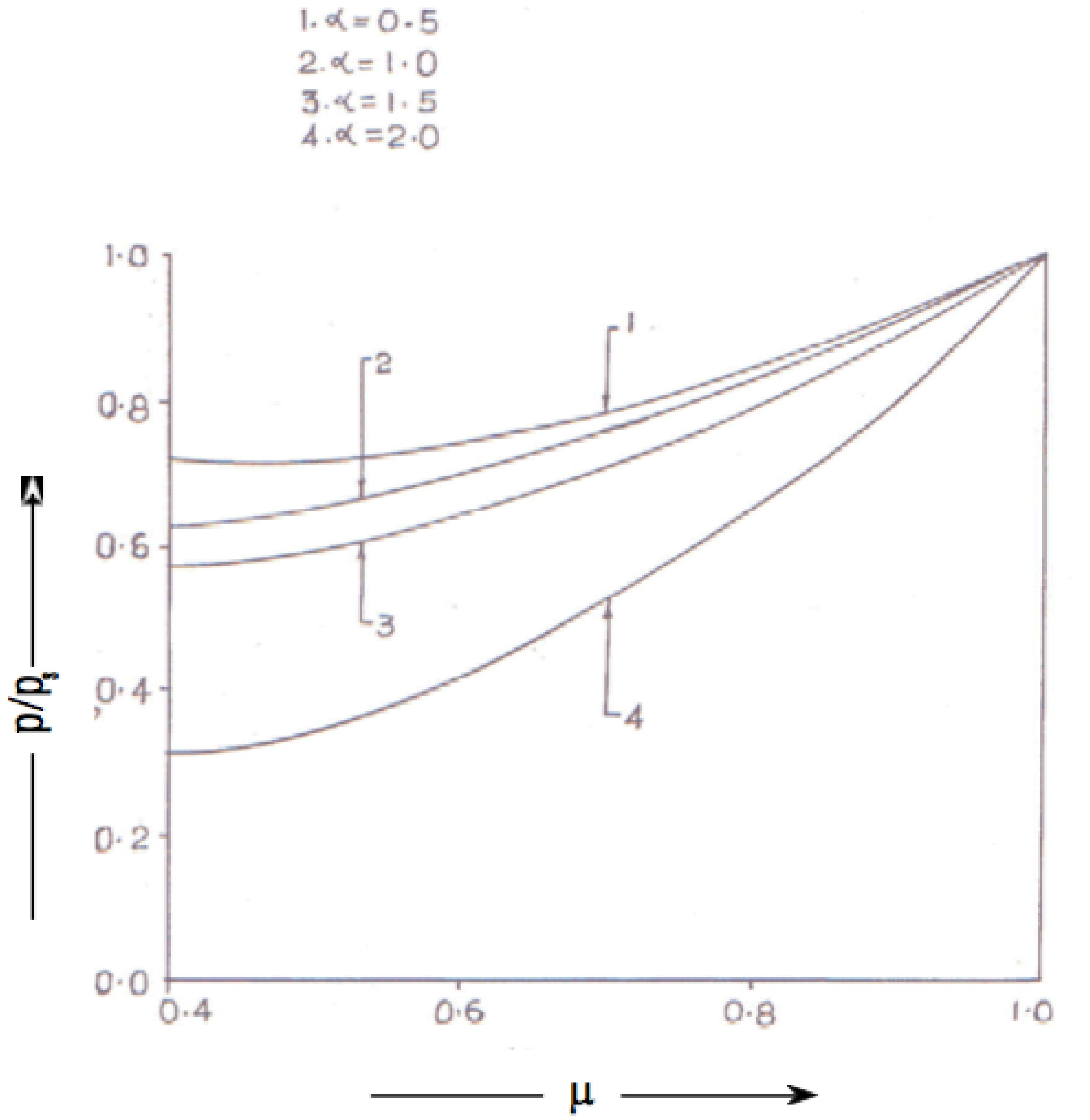

FIG - 2

PRESSURE DISTRIBUTIONS BEHIND THE SPHERICAL SHOCK WAVE AS FUNCTION OF μ FOR $\gamma = 5/3$

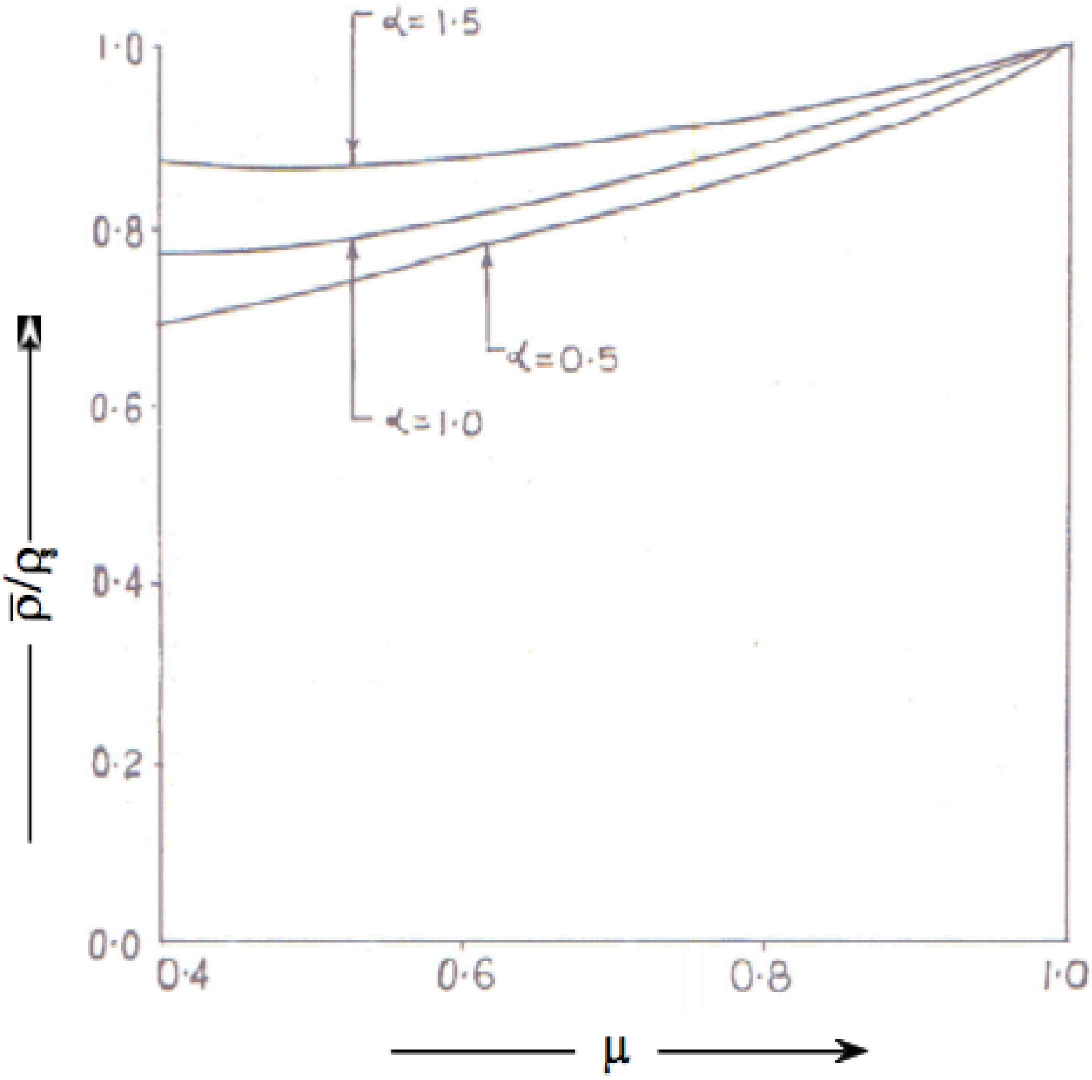

FIG - 3

DENSITY DISTRIBUTIONS BEHIND THE SPHERICAL SHOCK WAVE AS FUNCTION OF μ FOR $\gamma = 5/3$

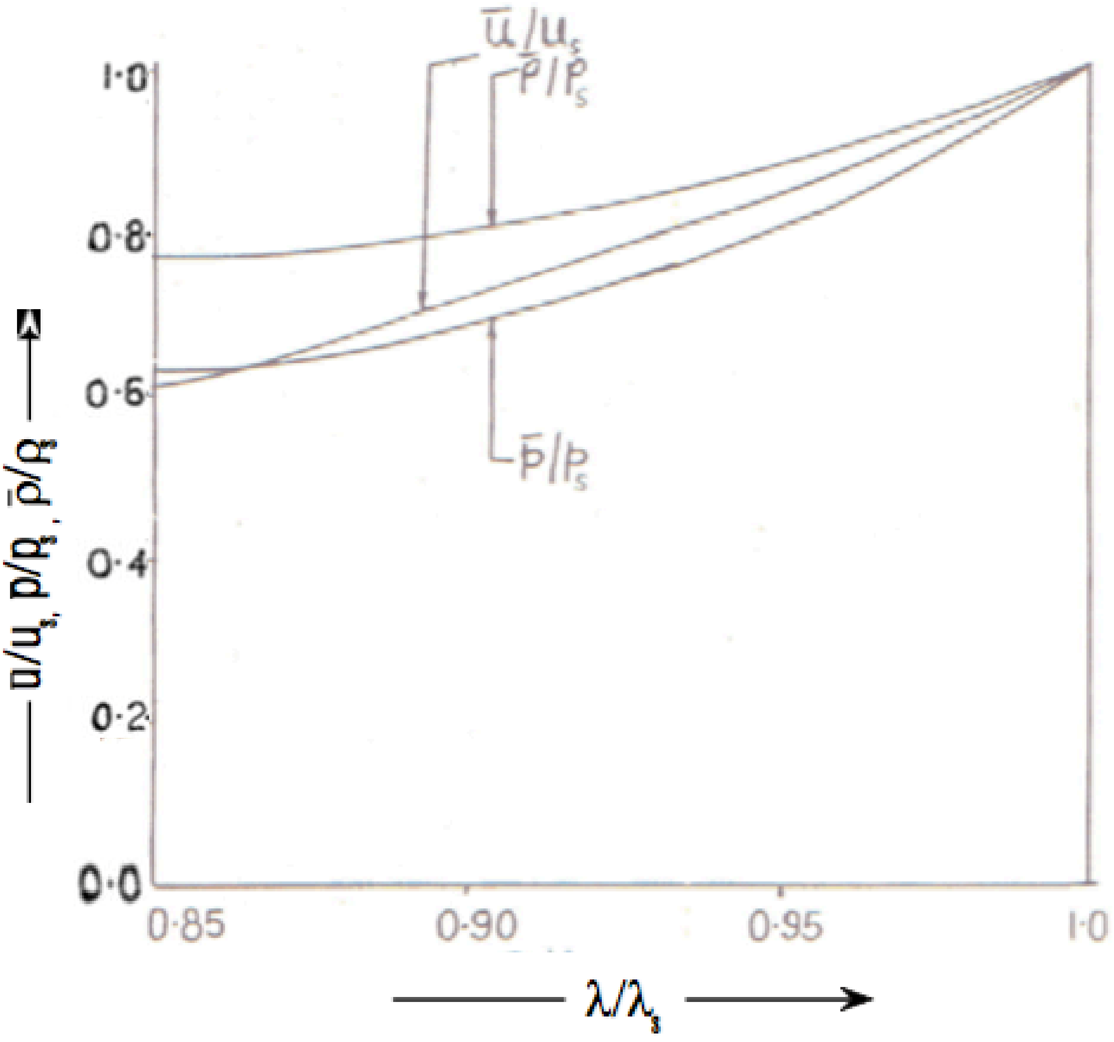

FIG - 4

FLOW VARIABLE DISTRIBUTIONS BEHIND THE SPHERICAL SHOCK WAVE AS FUNCTION OF RADIUS(r/R) FOR $\gamma = 5/3, \alpha = 0.5$

CHAPMAN-JOUGUET CASE

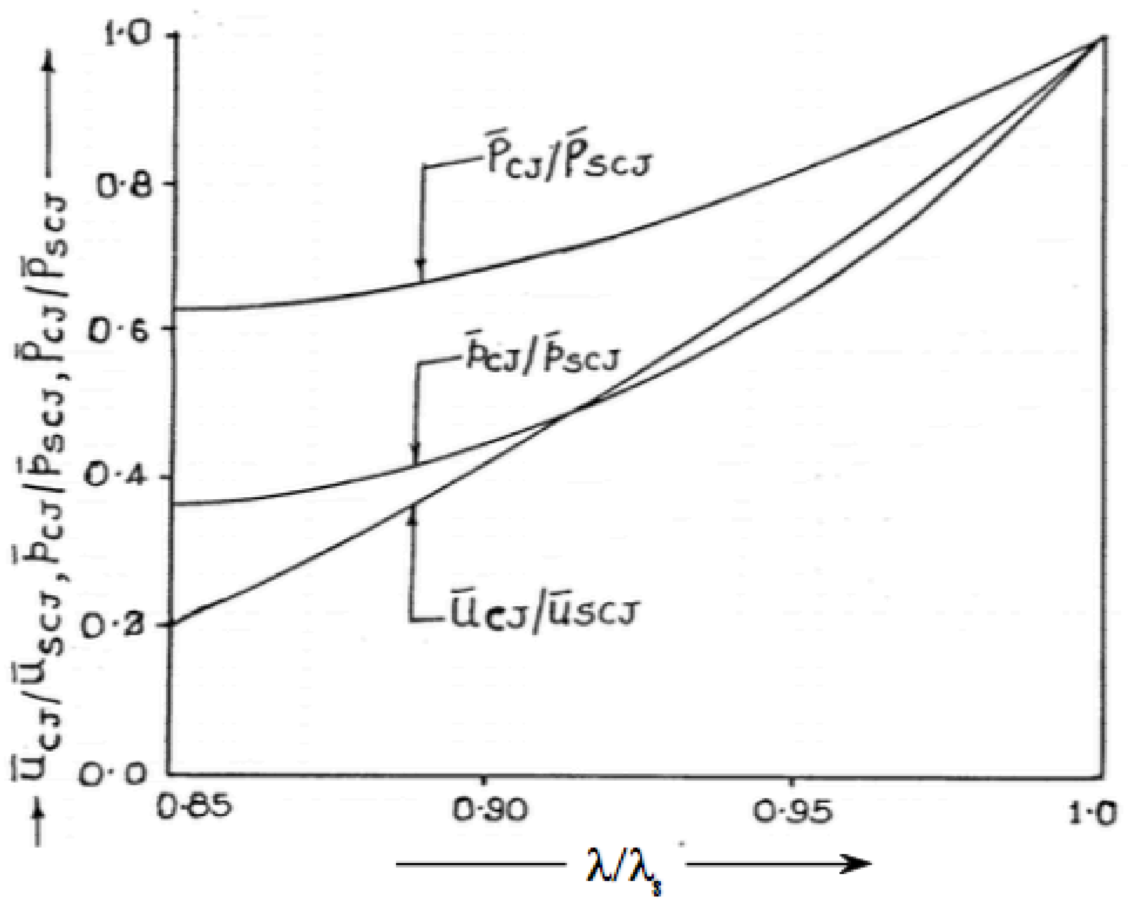

FIG - 5

FLOW VARIABLE DISTRIBUTIONS BEHIND THE SPHERICAL SHOCK WAVE AS FUNCTION OF RADIUS(r/R) FOR $\gamma = 5/3, \alpha = 1.5$

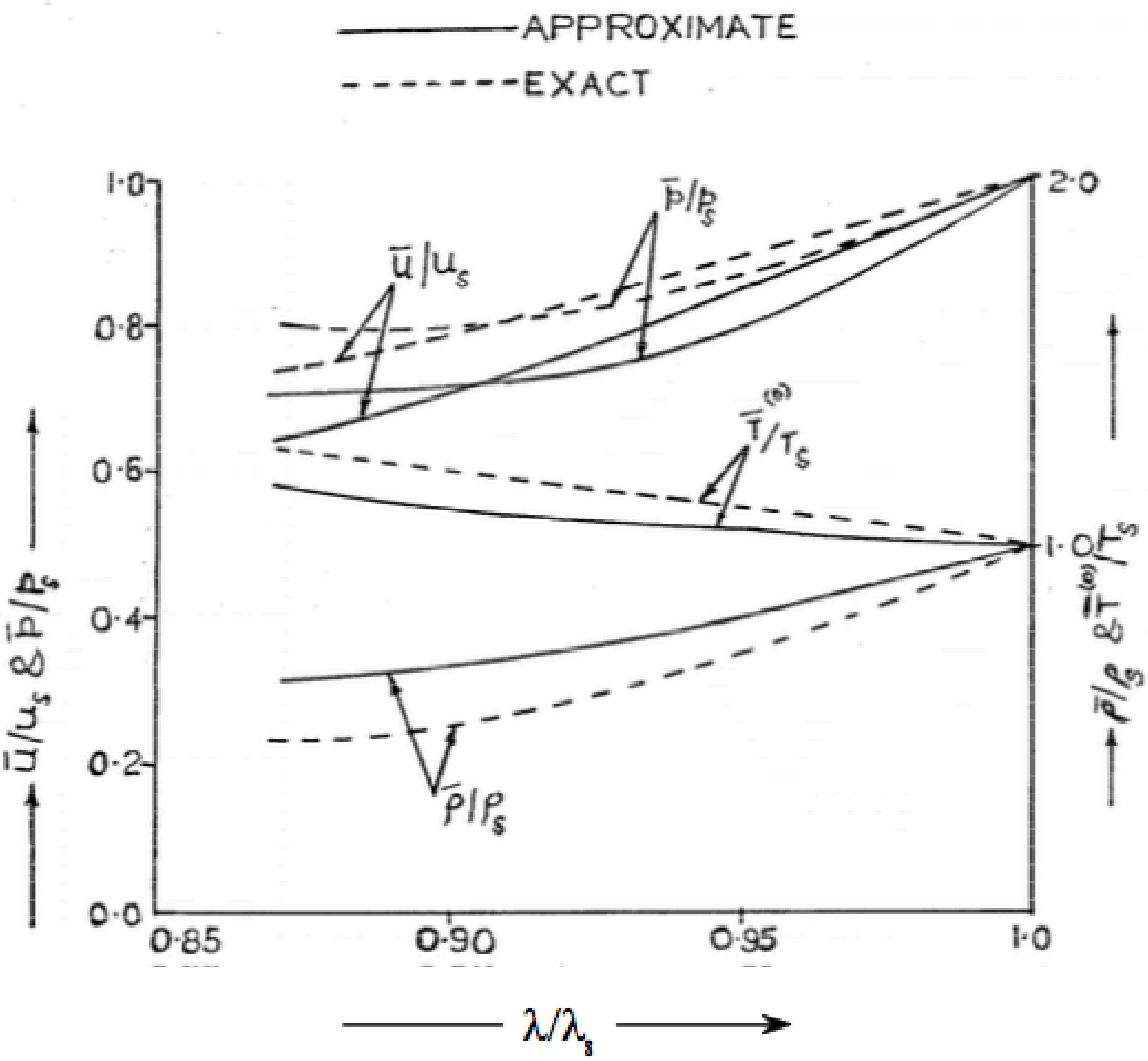

FIG - 6

FLOW VARIABLE DISTRIBUTIONS BEHIND THE SPHERICAL SHOCK WAVE AS FUNCTION OF RADIUS(r/R) FOR $\gamma = 5/3$

Equation (3.8) - (3.15) give the similarity variable λ_{cj}, the reduced particle velocity u_{cj}, pressure p_{cj} and density ρ_{cj}. The value of the dimensionless position of the shock wave is $\lambda_s = 1.067$ and the dimensionless position of Chapman-Jouguet shock is $\lambda_{scj} = 1.055$ for $\gamma = 5/3$.

The reduced flow variables are normalized using their values immediately down-stream of the shock layer and radial distance is normalized by the radial position of the shockwave. Figures 1, 2 and 3, show the distributions of normalized reduced particle velocity, reduced pressure and reduced density, respectively, as a function of μ for spherical case for $\gamma = 5/3$ and $\alpha = 0.5$, 1.0, 1.5, and 2. It is seen that the velocity and density decrease monotonically towards the center (axis). The pressure decreases behind the shock and then becomes constant. Fig. 4 show the distributions of normalized reduced particle velocity, pressure and density as function of μ for spherical shock for $\gamma = 5/3$ and $\alpha = 0.5$ only. Fig. 5 depicts the distributions of flow variables for the Chapman-Jouguet shock as function of normalized Eulerian similarity variable λ for spherical case for $\gamma = 5/3$ and $\alpha = 1.5$ only. Our solution gives accurate results and trend of the flow field behind the shock except for the values of μ less than $\mu = 0.4$. This may be due to the fact that self-similar solution is valid in a limited region, Zel'dovich and Raizer [3]. We recover the solution for radiation driven shock spherical and cylindrical shock, propagating in a uniform media by setting α equal to zero in the law of ambient density. Fig. 6 depicts the flow variable distributions behind the spherical shock propagating in uniform medium (α = o) and compares the analytic solution with the exact solution [8].

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