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The Computation of Octics Fields with Quadratic Subfields and Minimum Discriminants

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Abstract: We described the computation of no-primitifs octics fields with quadratic subfield and no quartic subfield, of disciminant less than $B = 3.10^9$ in absolue value. We find the minimum discriminant for all signatures and for all the possilble Galois group.

Keywords: Quadratic field, quartic field, relative extension, discriminant.

1. Introduction

Several works of constructed lists of number fields had established in different degrees up to 10, for example, in degree 3 [1-3], in degree 4 [4-7], in degree 5 [8-9], in degree 6 [10-14], in degree 7 [15-19], in degree 8 [20-23], in degree 9 [16, 24], and in degree 10 [25]. There are essentially two methods for construction of tables of number fields which allow us to enumerate all number fields, of targeted degree and signature, whose discriminants are bounded by given bounds. The first one is based on the geometry of the numbers which allowed us to find bounds for the coefficients of irreducible polynomial of which a root is a primitive element of the field. We have to distinguish the case of primitive extensions of the one of no-primitive extensions. In the first case [26] gives since 1956, 1957 the basic tool for doing this. If one considers no-primitive fields, the situation improves considerately, since, thanks to a relative version of the theorem used in the absolute case [26] due to J. Martinet [27], we can do all our work using relative data. The second method is based on the class field theory [28]. Recall that class field theory gives an explicit description of only abelian extension of a number field. Thus, for example, we can use the class field theory to compute all quadratic extensions of a number field [20]. In this paper we have followed the method of explicit construction of the relative extension described in [27].

This work is organized into five sections. In section 2 we presented notations and different formulas used in the next; in section 3 we gives details of computation which allows us to construct explicitly all the relative polynomials $P(X)$ of wich one of the roots θ defines an desired number field and in particular our octics fields. In section 4 we give how to compute the Galois group of our fields, and cited them with the same notations in [29]. We can after that expose the following problem, i.e., the question whether every finite group occurs as the Galois group of a field extension of Q. Complete results for permutation groups of small degree have so far only been published for degree up to 11 [30-31]. More than 10 years ago, Malle [32] completed the explicited realization of primitive non-solvable permutation groups of degree $d \leq 15$ as Galois permutation group up to this degree. J. Kluners and G. Malle gives an explicit Galois realization of transitive groups of degree up to 15 in [33] and a database for field extensions of the rational in [34]. The last paper contains polynomials for all transitive permutation groups up to degree 15, and the fields of minimal discriminant with given signature. We look at the case of transitive groups of degree 8 and then have compared our results with the others, more

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precisely, the minimum discriminant by Galois group possible and by signature. The readers can remarque that we arrived at the same minimum discriminant by Galois group and by signature. In the last section are presented the minimum discriminants of our octic fields accompanied by those found in [34].

2. Notations

If K is a number field of degree n and for signature (r,s), we denote by d_K its discriminant, by Z_K its ring of integers and by $J(K)$ the set of distinct Q-isomorphisms of K into C. For every c in K, we denote by $c^{(1)}, c^{(2)}, \ldots\ldots, c^{(r)}$ its real conjugates by $c^{(r+1)}, \ldots\ldots\ldots, c^{(r+2s)}$ its complex conjugates and we set $T_j(c) = \sum_{i=1}^{n} |c^{(i)}|^j$. Let $P(X) = X^m + a_1 X^{m-1} + \ldots\ldots + a_m \in Z_K$ defined a relative extension L/K of degre m where K is an number field of degre n' ($n = mn' = [L/Q]$). If $c^{(1)}, c^{(2)}, \ldots, c^{(n')}$ are the conjugates of c in K/Q, then $P = P^{(1)}, P^{(2)}, \ldots\ldots\ldots, P^{(n')}$ are polynomial with $a_i^{(h)}$ as coefficients. Let $f(X) = \prod_{h=1}^{n'} P^{(h)}(X)$ denotes the product of all conjugates of $P(X)$, which has integer coeffients and is either irreductible or a power of an irreductible polynomial. Let $\theta_1, \ldots\ldots, \theta_n$ the roots of $f(X)$ ordered so that $\theta_1, \ldots\ldots, \theta_m$ are the roots of $P(X)$ and $\theta_1^{(h)}, \ldots\ldots, \theta_m^{(h)}$ are those of $P(X)^{(h)}$. For each naturel number j we consider the power sums

$$s_j = s_j(\theta) = \sum_{i=1}^{m} \theta_i^j \text{ and } S_j = S_j(\theta) = \sum_{i=1}^{n} \theta_i^j \tag{2.1}$$

$$T_j(\theta) = \sum_{i=1}^{n} |\theta_i|^j. \tag{2.2}$$

Clearly, for $2 \leq j \leq m$

$$|S_j| \leq T_j(\theta). \tag{2.3}$$

Let $\delta_{L/K}$ be the relative discriminant of L over K, then the absolus discriminants d_L of L and d_K of K are related by the relation

$$d_L = (-1)^s |d_K|^m \mathcal{N}_{L/K}(\delta_{L/K}) \tag{2.4}$$

The following theorem given by J. Martinet [27], which is a generalization of the Hunter-Pohst theorem [26], is fundamental for the sequel.

Theorem 2.1. Let L be a number field of degree n, extension of degree m, of a subfield K of degree n'. There exists an integer $\theta \in L/K$ such that

$$\sum_{i=1}^{n} |\theta_i|^2 \leq \frac{1}{m} \sum_{h=1}^{m} |a_1^{(h)}(\theta)|^2 + \gamma_{n-n'} \left(\frac{|d_L|}{m^{n'} |d_K|}\right)^{\frac{1}{n-n'}} \tag{2.5}$$

where γ_j is the Hermite canstant in dimension j, which values are knowned for $j \leq 8$; we give it in Table 1:

Table 1

j	2	3	4	5	6	7	8
γ_j^j	$\frac{4}{3}$	2	4	8	$\frac{64}{3}$	64	256

3. Construction of the Tables

We are going to develop a method allowing to construct all these relative polynomials which defines our field L over K. By the condition $|d_L| \leq B$ and the formul (2.4) we deduce that we must interested only at the field K with $|d_K| \leq B^{\frac{1}{m}}$. To construct all the polynomials $P(X)$ of which one of the roots generates one of the fields L over K we work in the field $K = Q(\theta)$, we assume then that the discriminant d_K and an integral basis $\mathcal{V} = \{\nu_1 =; 1, \nu_2, \ldots\ldots\ldots, \nu_{n'}\}$ of K are already known. The theorem (2.1) shows that the coefficient a_1 of $P(X)$ may be chosen of the form $a_1 = \sum_{i=1}^{n'} \alpha_{1,i} \nu_i$. Since inegality (2.5) is valid by an translation by an element of Z_K, then we only have to take a_1 run through a system of representant of Z_K modulo mZ_K, and therefore only $m^{n'}$ values can be taken for a_1.(i.e $\alpha_{1,i} in\{0,\ldots\ldots,[\frac{m}{2}]\}$). Notice that for all value of a_1 there correspond a real constant bound $C(B) = \frac{1}{m}\sum_{h=1}^{n'} |a_1^{(h)}|^2 + M$ on $T_2(\theta)$. (where $M = (\gamma_{n-n'}^{n-n'} \frac{|d_L|}{m^{n'}|d_K|})^{\frac{1}{n-n'}}$). Then for the $m^{n'}$ possibles values for a_1, we chose only those for which $C(B)$ is minimal. Once a convenient value of a_1 is determined, we evaluate the remaining coefficients by induction with the help of Newton's formulas:

$$s_1 = -a_1 \text{ and } s_m = -ma_m - \sum_{i=1}^{m-1} a_i\, s_{m-i} \text{ for } 1 \leq m \leq n'. \quad (3.1)$$

where s_m denotes the relative symmetric functions defined by

$$s_m = Tr_{L/K}(\theta^m) \text{ for } m \in N.$$

The following theorem, due to Pohst [14], give a manner to compute upper bounds for the functions $T_m(\theta)$.

Theorem 3.1. Let T, N be positive constants satisfaying $(T/n)^{\frac{n}{2}} \geq N$. Then

$$T_m(x_1,\ldots\ldots,x_n) = \sum_{i=1}^{n} x_i^m$$

$m \in Z, m \neq 0$ and $m \neq 2$) has a global maximum on the set

$$S = \{x \in R_+^n\colon \sum_{i=1}^{n} x_i^2 \leq T, \prod_{i=1}^{n} x_i = N\}$$

at a point y with at most two different coordinates.

Remark 3.1. The condition $(T/n)^{\frac{n}{2}} \geq N$ is necessary since otherwise S is empty. For m = 2 the maximum of $T_m(x)$ is T.

Using theorem (3. 1) (where $T = C(B)$ and $N = 1$) we compute upper bounds for $T_q(\theta)$ for q = 3,..., m. Since we must have

$$\sum_{h=1}^{n'} |s_q^{(h)}| \leq T_q(\theta)$$

which comes from the inequalities

$$\sum_{h=1}^{n'} |s_q^{(h)}|^2 \leq (\sum_{h=1}^{n'} |s_q^{(h)}|)^2 \leq T_q(\theta)^2$$

we deduce possible values for coefficients of our relative polynomial P(X). For each of the constructed polynomials, we start by verifying if it define a field of desired signature , we eliminate those having too large values of $T_2(\theta)$, we test irreducibility of the polynomial P(X), we then compute the absolue discriminant and thus we elimine all field L for which the inegality $|d_L| \leq B$ is not verified. In the last test of isomorphism fields are practiced and then have our tables.We recall that in [1] there is specially functions for doing all calculs, in number fields, cited previously.

3.1 Octic Case with Quadratic Subfield and No Quartic Subfield

From now on, L denotes a octic field with quadratic field and no quartic field and of disciminant d_L less than $B = 3.10^9$ in absolue value. As we are concerned with the construction of lists of our octics fields and by (2.4) we must consider onlythe quadratic fields K whith d_K less than $B^{\frac{1}{4}}$ in absolue value. Each relative quartic extension of a quadratic field may be defined by a polynomial of degree 4 with coefficients in the subfield. In this section we develop a method of computation which allows us to construct explicitly all the relative polynomials

$$P(X) = X^4 - aX^3 + bX^2 - cX + d \in Z_K[X]$$

of which one of the roots θ defines our octic field such that $L = K(\theta)$. In this case theorem (2.1) becomes

Theorem 3.2. There exists an integer $\theta \in L/K$, such that $L = K(\theta)$ and

$$\sum_{i=1}^{8} |\theta^{(i)}|^2 \leq \frac{1}{4}\sum_{i=1}^{2} |Tr_{L/K_i}(\theta)|^2 + (\frac{4}{3}\frac{B}{d_K})^{\frac{1}{6}} \quad (3.2)$$

The coefficient $a \in Z_K = Z \oplus Z\,\omega$ where

$$\omega = \begin{cases} \frac{\sqrt{d_K}}{2} & if\ d_K \equiv 2,3 \pmod 4 \\ \frac{1+\sqrt{d_K}}{2} & if\ d_K \equiv 1 \pmod 4 \end{cases}$$

We have $a = a_1 + a_2\,\omega$ where $a_i \in Z$ modulo 4 for $i = 1,2$; Then there is 16 possibles values for a. ($a_i \in \{-1,0,1,2\}$). If we note $\mathcal{W}(K)$ the set of roots of unity of K, then if $d_K = -3$ ($d_K \equiv 1 \pmod 4$) $\mathcal{W}(\mathcal{K}) = \{\pm 1, \pm j\}$ then we have only 4 possible values for a and if $d_K = -4$ ($d_K \equiv 0 \pmod 4$) $\mathcal{W}(\mathcal{K}) = \{\pm 1, \pm i\}$ we have only 6 possibles values of a. For each of the value of a, we have possible values of C(B) which are bounds for $T_2(\theta)$ as explicated in previous section. Bounds for the the coefficient d are given by the inequality $|N(d)| \leq (T_2(\theta)/4)^4$ which follows from the inequality between arithmetic and geometric sums. The Newton's formulas in this case followed that $s_2(\theta) = a^2 - 2b$ and $s_3(\theta) = 3c + as_2(\theta) - ab$. Used method explained in the previous section which allows us to have bouds for $T_3(\theta)$ and then bounds for $s_2(\theta)$ and $s_3(\theta)$, finally we obtained possibles values for the cefficients b and c. We tested the irreducibility of polynomial $P(X)$, we estimated $\sum_{i=1}^{8} |\,\theta_i|^2$ which we compare with the bound $T_2(\theta)$ given by geometric methods. All this calculs was done by the logiciel Pari[1]. After all that, we must verified that L no contained an quartic subfield since if it is the case then we had $d_{K_4}^2$ must devided d_L where K_4 is an quartic subfield of L. At this time we find good polynomials.

4. Computation of Galois Groups

To compute the Galois group of each polynomial in our list we have used the Pari logiciel [35], then we obtain the Galois group as a strong generating set, whose elements are permutations on all roots of the given polynomial. The notations for the group names similar to that of Butlur and McKay [29]. In our case, after calculs we find only 7 groups possible noted respectively $T_{33}, T_{34}, T_{41}, T_{42}, T_{45}, T_{46}$ and T_{47} as same notations in [29]. We give in tableau 1 the order of each groups and its parity ("+" means that the group is of even permutations).

Table 2

Group	T_{33}	T_{34}	T_{41}	T_{42}	T_{45}	T_{46}	T_{47}
order	96	96	192	288	576	576	1152
parity	+	+	+	+	+		

We give the systems of generators for each group:

$T_{33} = <(1,7,3,5),(2,8,4,6),(2,4,3),(6,8,7)>$

$T_{34} = <>$

$T_{41} = <(1,7,3,5),(2,8,4,6),(2,4,3),(6,8,7),(1,8),(2,5),(3,6),(4,7)>$

$T_{42} = <(1,3),(2,4),(6,8,7),(1,5),(2,6),(3,7),(4,8)>$

$T_{45} = <(1,3),(2,4),(6,8,7),(1,5),(2,6),(3,7),(4,8),(1,8),(2,5),(3,6),(4,7)>$

$T_{46} = <(1,3),(2,4),(6,8,7),(1,6,2,5),(3,7),(4,8)>$

$T_{47} = <(1,2),(1,5),(2,6),(3,7),(4,8),(1,2,3,4)>$.

In Table 3, we give the number of the octics fields founded in the lists by signature and by the Galois group. We can remarqued that the Galois group change only in signatures 0 and 4, otherwise it is the T_{47}.

Table 3

	T_{33}	T_{34}	T_{41}	T_{42}	T_{45}	T_{46}	T_{47}
0	26	10	2291	2200	1238	52	1014749
2	-	-	-	-	-	-	6838
4	12	-	741	32	1325	376	15248
6	-	-	-	-	-	-	2560
8	-	-	-	-	-	-	48

5. Minimum Discriminants

Arrived at this step, we had a list of polynomials $f(X)$ wich defined octic number field with quadratic subfield and no quartic subfield , each of them with his signature, Galois group and discriminant less than 3.10^9 in absolute value. We than had tried this tables by signature, we obtain 5 tables, after that we had classed the fields by disciminants croissants and in this way found the minimum discriminant of our fields by signature. In [33], J. Kluners and G. Malle give the Galois realisation of transifs groups whose degres less than 15. In this paper, the author describes methods for the construction of polynomials with certain types of Galois groups, and deuced that all transitive groups G up to degree 15 occur as Galois groups of regular extension of Q and for each compute a polynomial $f(X) \in Q[X]$ with Gal(f) = G. In Tables 4, 5, and 6, we presented these minima, * are found in [34].

5.1 Minimum Discriminant in Signature 0

In Table 4, we give from left to right the group the Galois of our octic field L, it's minimum Disciminant and the polynomial $f(X)$ defined L/Q in signature 0.

Table 4

g.Galois	minimum D_L	$f(X)$
T_{33}	1262025625 1262025625 *	$X^8-4X^7+7X^6+5X^5-38X^4+23X^3+31X^2-36X+36$ $X^8-2X^7-4X^5+12X^4+2X^3-14X^2-5X+11$ *
T_{34}	1614110976 1614110976 *	$X^8-4X^7+10X^6-20X^5+26X^4-8X^3+16X^2-4X+4$ $X^8+6X^7+12X^6+10X^5+24X^4+42X^3+40X^2+18X+3$*
T_{41}	24255625 24255625 *	$X^8-X^7+X^6+3X^5+X^4-7X^3+6X^2-X+1$ $X^8-3X^7+4X^6-4X^5+6X^4-6X^3+4X^2-2X+1$ *
T_{42}	12075625 12075625 *	$X^8-3X^7+3X^6+2X^5-3X^4-4X^3+11X^2-10X+4$ $X^8-2X^7+2X^6-2X^5+2X^4-X+1$ *
T_{45}	55115776 55115776 *	$X^8-4X^7+8X^6-20X^5+41X^4-44X^3+26X^2-8X+1$ $X^8-2X^6+X^4+4X^2-4X+1$ *
T_{46}	27653125 27653125 *	$X^8-4X^7+11X^6-23X^5+29X^4-34X^3+49X^2-25X+25$ $X^8+2X^7+7X^6+11X^5+19X^4+20X^3+20X^2+10X+5$*
T_{47}	1342413 1342413 *	$X^8-3X^7+6X^6-7X^5+7X^4-6X^3+4X^2-2X+1$ $X^8-2X^5+X^4+3X^3-2X^2-X+1$ *

Table 5

g.Galois	minimum D_L	$f(X)$
T_{33}	2522550625 2522550625 *	$X^8-2X^7+6X^6+2X^5-26X^4+24X^3-24X^2+3X-11$ $X^8-3X^7-X^6+9X^5-11X^4+11X^3-13X^2+21x-9$ *
T_{41}	258405625 258405625 *	$X^8-4X^7+3X^6-2X^5+16X^3+14X+1$ $X^8-3X^7+4X^6+11X^5-19X^4-X^3+14X^2-7X+1$ *
T_{42}	618765625 618765625*	$X^8-4X^7+4X^6-3X^5-11X^4+4X^3-9X^2-2X+1$ $X^8-X^7-X^6-2X^5-X^4+X^3-4X^2+7X+1$ *
T_{45}	118810000 118810000 *	$X^8-2X^7-X^6+X^5-2X^4+15X^3-17X^2+7X-1$ $X^8-X^7-4X^6+4X^5+7X^4-8X^3-4X^2+5X-1$ *
T_{46}	402753125 402753125 *	$X^8-4X^7+2X^6+8X^5-11X^4+9X^3-7X^2+2X-1$ $X^8-X^7+X^5-4X^4+5X^3+6X^2-2X-1$ *
T_{47}	16643125 16643125 *	$X^8-4X^7+2X^6+8X^5-11X^4+9X^3-7X^2+2X-1$ $X^8-X^7+X^5-2X^3-2X^2+X+1$ *

Table 6

Sign	Minimum D_L	$f(X)$	Galois group
0	1342413	$X^8-3X^7+6X^6-7X^5+7X^4-6X^3+4X^2-2X+1$	T_{47}
2	-5756875	$X^8-2X^6+3X^5-3X^3+2X^2+X-1$	T_{47}
4	16643125	$X^8-4X^7+2X^6+8X^5-11X^4+9X^3-7X^2+2X-1$	T_{47}
6	-74906875	$X^8-3X^6-3X^5+3X^4+7X^3-2X^2-3X+1$	T_{47}
8	661518125	$X^8-2X^7-7X^6+13X^5+13X^4-22X^3-8X^2+X+1$	T_{47}

5.2. Minimum Discriminant in Signature 4

Table 5 presented the same results at Table 4 but it concerned signature 4.

5.3. Minimum Discriminant All Signatures

We finish this work in giving the minimum disciminant D_L of a octic field L with quadratique subfield and no quartic subfield by signature. We resumed the results in Table 6 and we can remarqued that the Galois group corresponded at the minimum D_L is T_{47} in all signatures.

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