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A Straightforward Model Eliciting Activity and the Power of “What If?” in Supporting Students’ Higher-Order Thinking

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Abstract: A Model Eliciting Activity (MEA) is one which explicitly reveals student thinking processes through their descriptions, explanations, justifications, and representations throughout the modeling activity [1]. Lesh and Caylor [2] enumerate principles that are necessary for a task to be recognized as an MEA. This paper uses a straightforward MEA, the trapezoidal tables problem, to examine student solutions that demonstrate the principles of an MEA, and reveal student thinking, as well as to discuss “What If?” scenarios that extend student thinking and generate predictions of how changing problem parameters will modify student solutions.

Key words: model eliciting activity, higher order thinking, taxonomy

1. Introduction

Mathematical modeling is a cornerstone of mathematical thinking. The process of creating a mathematical model represents one of the key steps in what Polya (1957) identified as “the problem-solving process” [3]. Polya’s four steps (understand the problem, make a plan, do the plan, and look back) offer a widely-adopted heuristic that enables students to begin to understand the process of mathematics, and, incidentally, a reason for learning all that mathematical content. Teachers need to present students with situations that encourage the creation of a model as a potential route to problem solution. It is important that the situations offered to students be engaging, realistic, connected to students’ lives, and sufficiently challenging.

What makes a good mathematical modeling task? In our view, the quality of the task is determined by the behaviors of the students who engage in the task. This paper describes the administration of a model eliciting activity (MEA) to two grade 9 classes, a total of 50 students, in the fall of 2013. The task was chosen because it lends itself to multiple solution methods, including tables, graphs, equations, diagrams, and physical models using manipulatives. The MEA is quite straightforward. However, the critical piece of this investigation was the “What If?” questions that followed the basic task, encouraging students to conjecture, predict, and generalize.

While there were no exemplars specific to this task, students were familiar with the modeling process, having experienced a number of other modeling activities during the semester. The students were allowed up to a full class period (75 minutes) to complete the task, individually or in pairs. The teacher acted in the role of facilitator, providing the level of support and scaffolding normally seen in the class. As much as possible, students were encouraged to provide peer support for each other.

2. What Is a Model?

The literature treats modeling in two distinctive ways. For example, as a noun, “purposeful mathematical descriptions of situations, embedded in particular systems of practice” [4, p. 109] or “a variety of representational media, which may involve written

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symbols, spoken language, computer-based graphics, paper-based diagrams or graphs, or experience-based metaphors" [5, p. 159]. Other researchers focus on modeling, a verb: "the process of translating between the real world and mathematics in both directions" [6, p. 45]. Bonotto (2010) defines modeling through the steps required, "a process whereby a situation has to be problematized and understood, translated into mathematics, worked out mathematically, translated back into the original (real-world) situation, evaluated, and communicated" [7, p. 20]. These steps echo the modeling cycle proposed by Blum and Ferri (2009), emphasizing the need for a model to be related back to the original, real world situation that gave rise to the modeling activity, with evaluation and justification. Temur (2012) embeds modeling as a form of problem solving as "conceptual systems to explain and define mathematical concepts, tools, relations, actions, forms and settings" [8, p. 83]. This is similar to Lesh and Harel (2003) [5], who view modeling as a part of problem solving, with the purpose of describing or explaining other systems. In keeping with our focus on processes, we use the action oriented form of modeling, where the modeling process is seen as more important than the model, and the actions of representation, analysis, explanation, justification, evaluation, and communication are primary.

This focus on process illustrates the clear links from modeling to the Common Core Mathematical Practices [9]. Specifically, the Mathematical Practices most closely linked to modeling are: (a) make sense of complex problems and persevere in solving them, (b) reason quantitatively, (c) look for and make use of structure, and (d) construct viable arguments. The other Mathematical Practices, namely, attend to precision, look for and express regularity in repeated reasoning, and make strategic decisions about the use of technological tools, are also important, and may be related to the modeling process as the model is formulated and utilized. The Ontario Canada mathematics curriculum specifies seven Mathematical Processes: problem solving, communicating, representing, connecting, reasoning and proving, reflecting, and selecting tools and strategies [10-11]. Modeling is connected to all of these processes, with emphasis on the first five. The student work samples for the Trapezoidal Tables task demonstrate both the Mathematical Practices and Mathematical Processes.

The process of modeling requires students to mathematize, that is, to quantify, organize, dimensionalize, systematize, and coordinate [5]. In addition, modeling asks students to represent, explain, justify, analyze, evaluate, and communicate. These are all foundational skills for learning.

3. Reasons for Modeling

There are a number of diverse reasons for engaging in mathematical modeling. Modeling is a means to address both the social and individual perspectives on the nature of mathematics [4]. Blum and Ferri (2009) see mathematical modeling as a critical component of mathematical literacy, which they define as "the ability to deal with real world situations in a well-founded manner" [6, p. 47]. They identify four important functions of modeling: (a) to help students understand the world; (b) to develop mathematical competencies and attitudes; (c) to build an adequate picture of mathematics; and (d) to support learning of concepts, comprehension, motivation, and retention. Mason, Burton, and Stacey [12] envision three mathematical worlds: (a) a material world of confidently manipulable objects, including physical objects, images and symbols; (b) a world of mental imagery; and (c) a world of abstract symbols not yet immediately manipulable. They see modeling as linking the material world to the other worlds, especially the abstract, algebraic world. Modeling allows the development of objects in the abstract world to the level of confidence necessary for them to enter the material world. Forbus, Carney, Sherin, and Ureel II [13] identify three dimensions of importance for mathematical modeling. These are: to help understand the world, the modeling process itself, and practice using formal representations. As mentioned earlier, we focus on the second of these, the process of modeling. Teaching mathematical modeling provides students with a critical tool to interpret and understand reality [7]. This stance was recognized in the work of the Harvard Balanced Assessment Project, where model/formulate is seen as a critical element of the four-part problem-solving process (model/formulate, transform/ manipulate, infer/conclude, communicate/look back) [14].

4. Model Eliciting Activities

A Model Eliciting Activity (MEA) is one which explicitly reveals student thinking processes through their descriptions, explanations, justifications, and representations throughout the modeling activity [1]. Lesh and Caylor [2] enumerate principles that are necessary for a task to be recognized as an MEA: personal meaningfulness (reality, related to the student's life), model construction (there is a clear need for a model), self-evaluation (students judge usefulness), model generalizability (sharability and reusability): model documentation (the need for students to reveal their thinking), simplest prototype (the situation is as simple as possible while still requiring a model). A discussion of how our task addresses these six criteria is given later in this paper.

Simplest prototype is, in our opinion, the most difficult principle to address when creating modeling tasks. Lesh and Harel (2003) point out that, in general, it is difficult for students to make useful symbolic descriptions of physical situations; descriptions and explanations (or constructions) are not just relatively insignificant accompaniments to "answers". They are the most critical components of conceptual tools that need to be produced [5, p. 159]. Because models focus on structural characteristics of a situation, rather than physical or other characteristics [2, 5], it is often difficult for students to identify the important dimensions, while ignoring other, less relevant ones. This is compounded by the problem of transfer, wherein the portability of skills and knowledge to new or different situations is problematic. In school, most situations involve near transfer, with temporal proximity, as well as a high degree of similarity in problem specifications [15].

If all of the above principles are satisfied, the task can be identified as an MEA. The authors of this paper independently developed a very similar list prior to doing the literature review. Doerr and English [1] add two additional criteria, namely, explicit links to existing knowledge, and requiring high level thinking and reasoning. The link to existing knowledge is critical since students cannot be expected to enter into a modeling situation for which they have no relevant prior knowledge, or for which specialized knowledge is required to understand, model, or interpret a situation.

MEAs, which involve simulations of real life, can be distinguished from Guided Discovery Activities (GDAs), in which structured questions maneuver students towards and along a hypothetical learning trajectory, with a specific end in mind [2]. For example, an activity called penny circles asks students to investigate the number of pennies needed to (a) encircle a given circle around its circumference, and (b) fill up the circle. While an interesting and engaging activity, penny circles is a GDA. The expected outcomes are known to the teacher, and the investigation is structured to move students toward that outcome. This is clearly different than an MEA, which asks students to abstract the important elements of a situation, translate into mathematics, manipulate the mathematics, and then translate back to reach a supported solution to the real world problem.

A quality modeling task should also have elements of choice and, therefore, generate student interest. The task requires clear, explicit instructions, and both teacher and students should know what a quality product looks like (e.g., via a rubric or exemplars).

5. Higher Order Thinking Skills

Higher order thinking skills (HOTS) are one of the dimensions of deep learning, together with integrative learning and critical reflection [16]. In deep learning, students make connections and integrate knowledge into internal cognitive networks. Deep learning can be contrasted with surface learning, which focuses on facts and basic procedures [16]. HOTS can be contrasted with lower order thinking skills (LOTS), which consist of basic recall of facts or procedures, or application of a known procedure in a known situation. A specific situation may require LOTS or HOTS, depending on the learner's prior knowledge [17]. For example, a problem that can be solved using the sine law may be LOTS, if the learner has seen similar problems before, or the problem may require HOTS, if the problem situation is new to the learner.

The literature has a myriad of definitions for higher order thinking. Many of these definitions differ in their treatment of critical thinking or problem solving. In a literature overview, Lewis and Smith (1993) outline a number of definitions in which critical thinking and problem solving are considered sublevels of HOTS, while other definitions consider HOTS, critical thinking, and problem solving as differing macro-level constructs [17]. Fitzpatrick et al. [18] assert that HOTS and critical thinking are similar cognitive processes, but are not identical. Lewis and Smith (1993) offer this definition of HOTS: "Higher order thinking occurs when a person takes new information and information stored in memory and interrelates and/or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situations" [17, p. 136].

HOTS are sometimes identified using a taxonomy of learning, such as Bloom's taxonomy [19]. This taxonomy has six levels, Knowledge, Comprehension, Application, Analysis, Synthesis, and Evaluation, arranged hierarchically from the lowest (Knowledge) to the highest (Evaluation). The first two levels are usually considered LOTS, while the last three levels are usually identified as HOTS. Treatment of the Application level is inconsistent in the literature. Some studies [20] consider Application as HOTS, some (e.g., [21]) as LOTS, and some (e.g., [22]) consider Application as either LOTS or HOTS, depending on context. The linking of original Bloom to higher order thinking skills has not been without issues, however. Thompson [23], in a study of 32 high school mathematics teachers in the United States, found that a majority of teachers, even after training, could not identify a HOTS question using Bloom's taxonomy, and could not create one. In addition, the teachers often used their own definitions of higher order thinking skills rather than Bloom.

HOTS are treated similarly in revised Bloom [24]. Typically, the identification of HOTS in the Apply level requires examination of the sublevels of revised Bloom (Figure 1), to determine whether the situation is familiar or new to the learner. For example, the sublevel of Apply, Implementing, is "applying a procedure to an unfamiliar task" [24, p. 67]. This would be classified as HOTS, as well as the sublevels in Analyze (differentiating, organizing, attributing), Evaluate (checking, critiquing), and Create (generating, planning, producing) [24, pp. 56-57].

Both Bloom and revised Bloom consider only the cognitive domain. Marzano's New Taxonomy [25-27] identifies somewhat different cognitive levels, and adds consideration of metacognition and the self system (see Figure 2). HOTS would include all the sublevels of the Metacognitive system, all sublevels in the Cognitive domain of Knowledge Utilization, and the sublevels Generalizing and Specifying of the Cognitive domain Analysis. The sublevel Specifying refers to predicting, and may include formulating a hypothesis. Formulating hypotheses will also fall into the Knowledge Utilization categories of Experimenting and Investigating.

Teaching for HOTS was positively correlated with increased student achievement, while teaching for both LOTS and HOTS had the greatest impact [21]. Instructional strategies that promote HOTS include cooperative learning, graphic organizers, and think aloud [23, 28]. Marshall and Horton [29], in a study involving 22 middle school science and mathematics teachers found that teacher-facilitated inquiry-based instruction was correlated with development of HOTS in students.

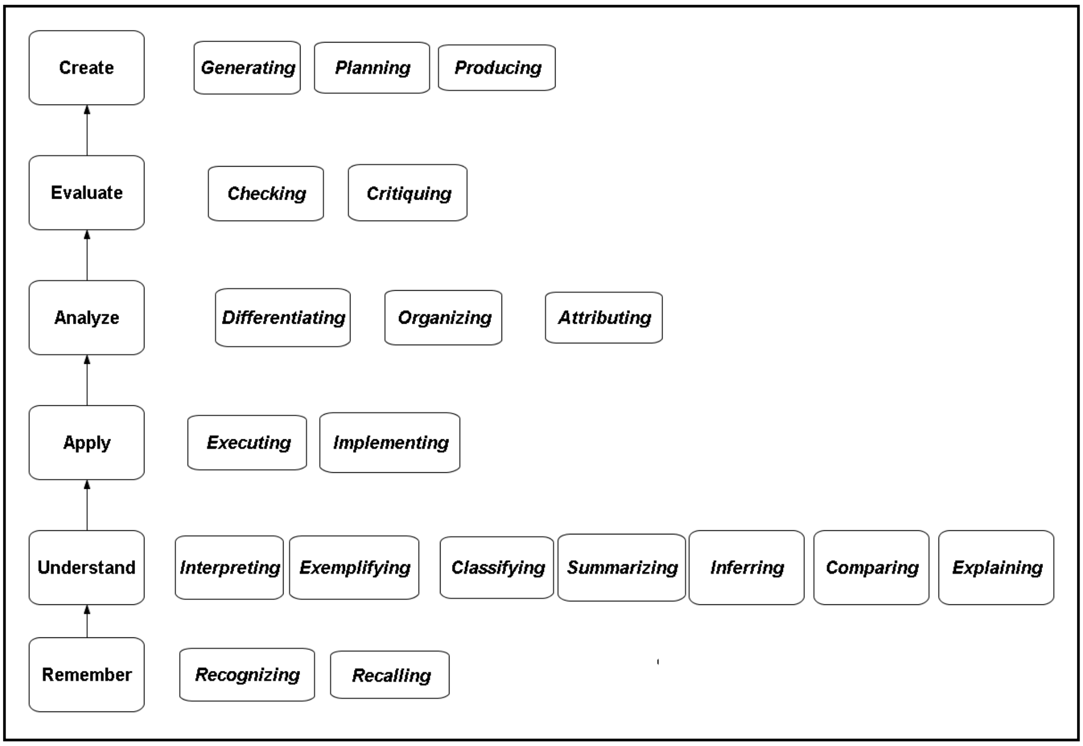


Fig. 1 **Revised Bloom showing sublevels. Constructed from information in [24]: A taxonomy for learning, teaching, and assessing.**

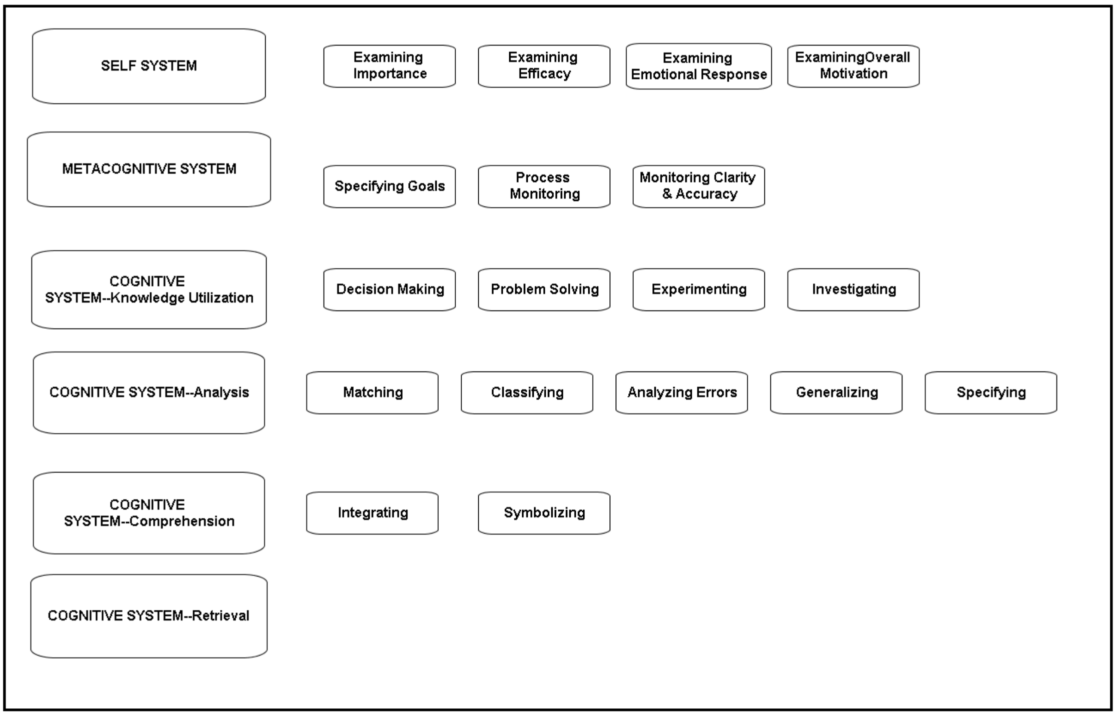


Fig. 2 **Marzano's New Taxonomy with Sublevels. Reproduced with permission from [26]: The new taxonomy of educational objective.**

6. Methodology and Method

The authors of this paper conducted several brainstorming sessions to identify an appropriate model eliciting activity. The task needed to satisfy the six criteria of Lesh and Caylor [2], and particularly meet two criteria identified as key by the author team: the basic task had to be straightforward and clearly demonstrate a need for a model as a solution technique, and the task had to have the potential to generate “What If?” questions as extensions to the basic task. After considering and discarding a number of possible tasks, we settled on the trapezoidal tables task (Figure 3).

This task was given to two grade 9 classes, a total of 50 students, in the fall of 2013. To ensure that the task was given in a genuine context, the task was presented by the classroom teachers, without any of the author team present. This helped keep the task in the usual realm of the classes, and reduced the potential for cross contamination by the authors. The task was presented in the first part of a 75 minute class. Students were allowed to work individually or in pairs, according to their own personal preference. Manipulatives were available for use, as well as graphing calculators. After the students completed the task, a whole-class discussion of “What If?” scenarios occurred. Initially the teachers posited some scenarios, and led a discussion of possible consequences for modifications to the basic solutions. As the discussion progressed, students began to pose scenarios, which were then analyzed by the rest of the class. The authors coded and sorted student work using inductive content analysis [30]. Selected student work samples are discusse below.

Subsequently, the authors interviewed the teachers, with a focus on the “What If?” scenarios, and transcribed the interview notes. These notes were then sorted using content analysis to identify commonalities and exceptionalities. Particular emphasis was given to correlating the results of the investigation to the MEA criteria [1-2].

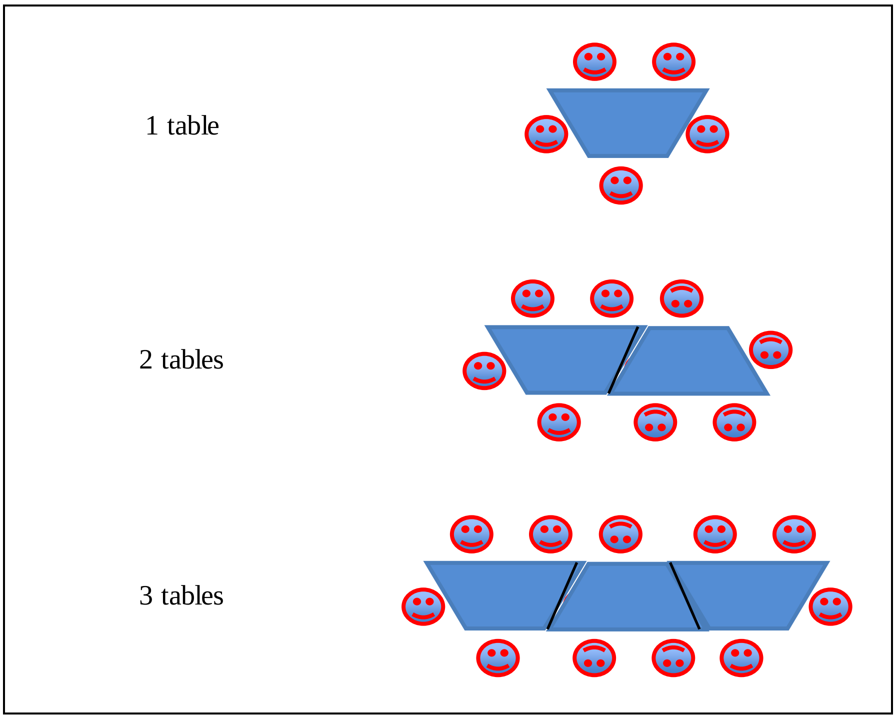


Fig. 3 The trapezoidal table problem: Sally set up trapezoidal tables as shown above. The diagrams show how many people can sit around one table, two tables and three tables. Sally says 39 people can sit around 12 tables that are pushed together like the pictures shown. Is she correct? Justify your choice. Show your thinking and your work.

7. Student Work Samples

Content analysis was used to sort student solutions into three groups: (a) predominantly graphical, (b) predominantly tabular, and (c) algebraic. A typical graphical solution is shown in Figure 4. This was the most common solution method chosen by the students. This may be the result of the teachers providing graph paper when the task was introduced. Although manipulatives were available for student use, none chose to make physical models or additional diagrams. This was quite probably due to the task being a simplest prototype, and most students found it to be straightforward. Possibly due to the timing of this research (late in the semester), many students chose to treat the task as a review or extension of concepts already dealt with in class.

Following is a discussion of four student work samples in more detail.

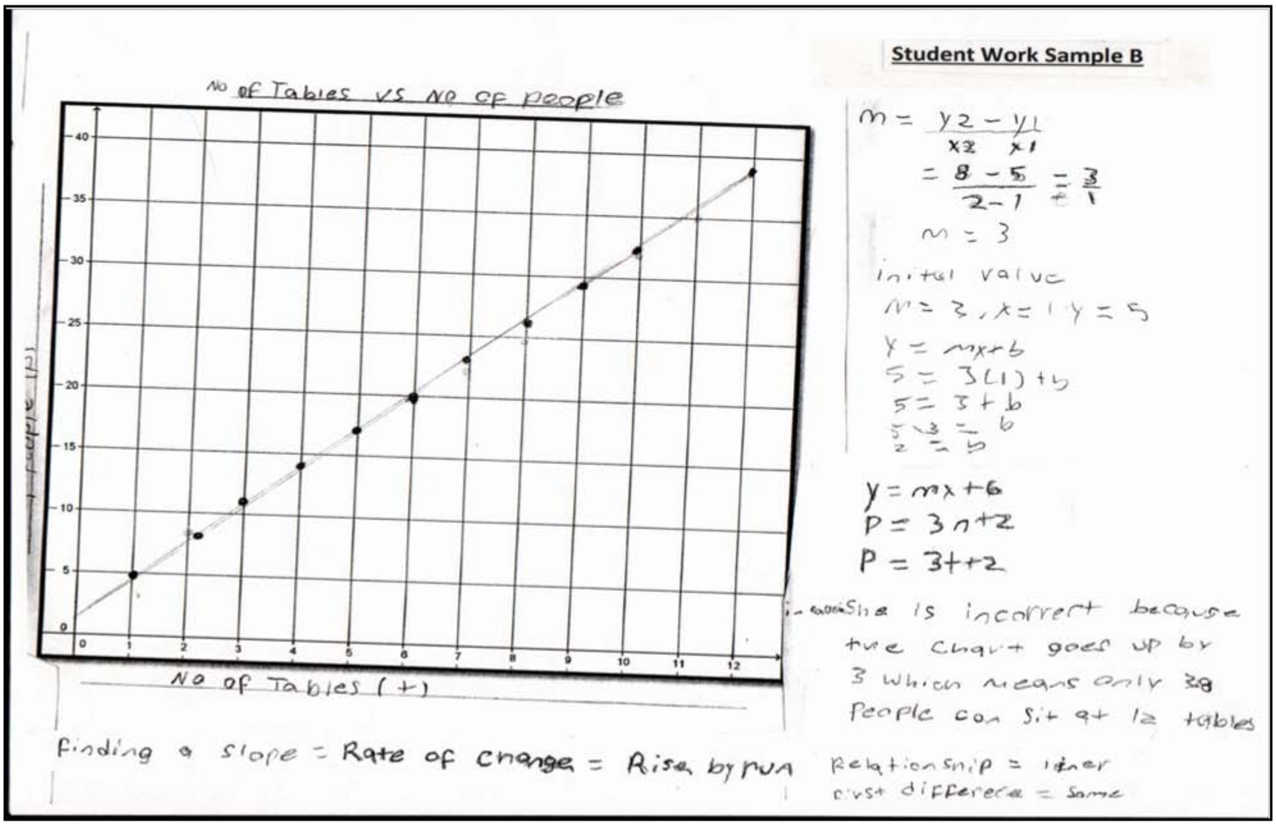
Sample H (Figure 5) is a well-articulated solution in which the student appears to be treating the problem as a routine exercise based on previously learned work. This sample (and several others) includes a discussion of partial variation, although the problem statement makes no reference to variation. This solution does not include a justification for the conclusion, although the student attempted to justify his/her conclusion by including a graph (not shown).

Sample E (Figure 6) shows good logic, based on the table that was created. The student includes a conjecture that Sally made an addition error to reach her (incorrect) conclusion, but offers no justification for either the conjecture about Sally's mistake or her own conclusion. Again, justification was offered through inclusion of a graph (not shown).

In Sample C (Figure 7), the student recognized that justification was needed. They offer a series of substitutions, with the statement "we tried it with multiple different table numbers and it worked". However, there is no discussion of what "worked" meant. It may have indicated that the results matched entries in a table, or that results matched the physical situation for different numbers of tables. If this information had been included, linking the algebraic information to the physical situation, this would have provided strong justification for their conclusion.

Sample J (Figure 8) is interesting because of the false start (scratched out work at top of page). This was potentially a very insightful solution, concluding that to have 39 people, more than three people would have to be added for each table added. However, the student reverted to a more standard solution, including a comment on partial variation. The "justification" was a verbal description of the algebraic equation.

In a follow-up discussion, the teachers involved indicated that everyone was engaged in this activity and most were able to formulate a solution, often based on previous work. They noted that justification was the weakest part of the solutions, usually reverting to alternate representations such as graphs. This may have occurred due to the statement in the problem "Justify your choice," which asked them to justify their conclusion, but not necessarily justify the choice of model or relate the model to the physical situation. In summary, the students were able to understand the physical situation, construct a mathematical model or models of the situation, analyze the models to formulate mathematical solutions, translate back to the physical situation, evaluate their results, and justify their methods and conclusions. In general, the justifications were disappointing. Very few justified their solution method by linking back to the physical situation, other than to answer Sally's question. Many students presented multiple solution techniques, generally a graph plus at least one of a table or equation.



Student Work Sample B

$m = \frac{y2 - y1}{x2 \quad x1}$

$= \frac{8-5}{2-1} = \frac{3}{1}$

$m = 3$

initial value

$m = 3, x = 1 \; y = 5$

$y = mx+b$

$5 = 3(1)+b$

$5 = 3+b$

$5-3 = b$

$2 = b$

$y = mx+6$

$P = 3n+2$

$P = 3t+2$

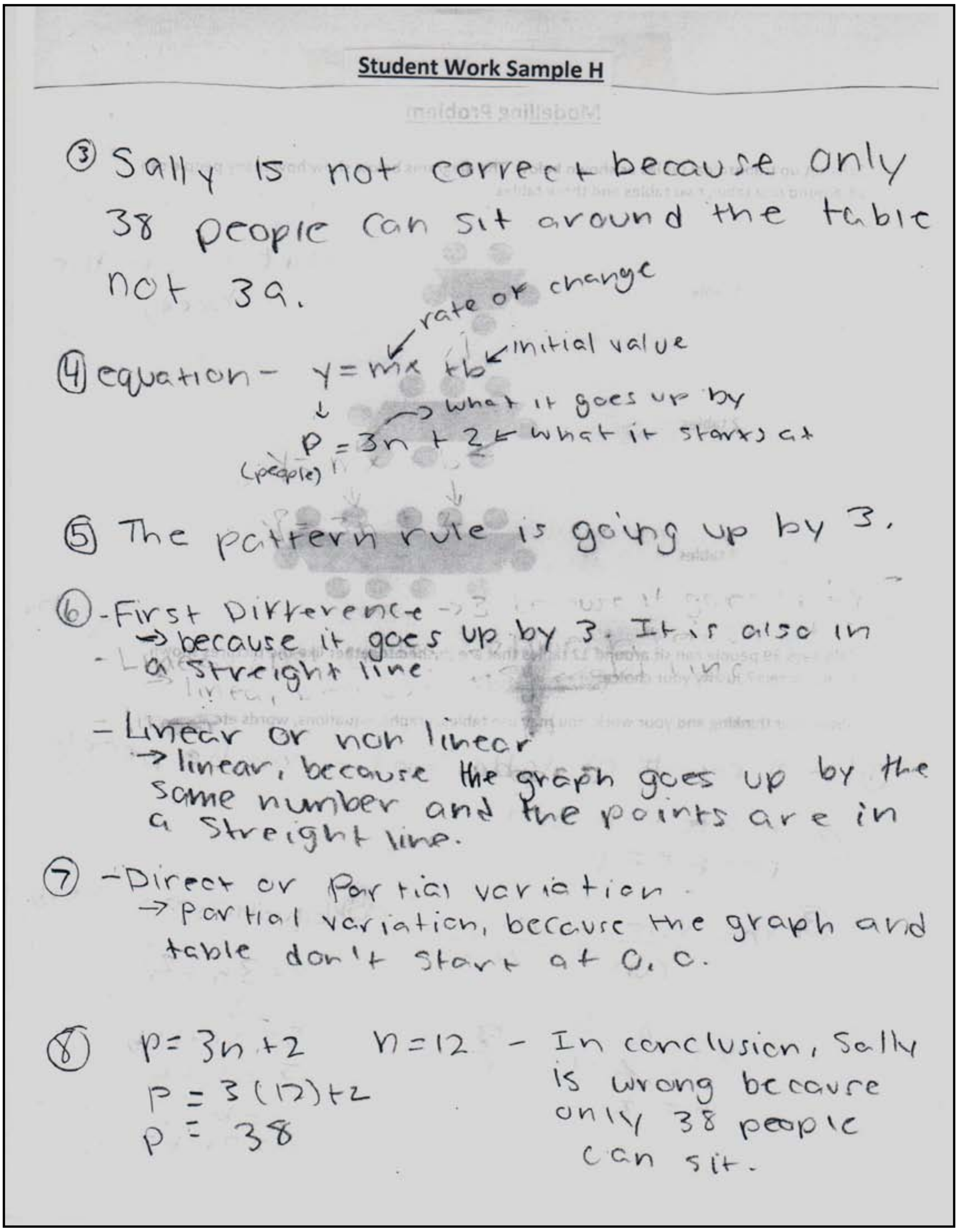
She is incorrect because the chart goes up by 3 which means only 38 people can sit at 12 tables

finding a slope = Rate of change = Rise by run

Relationship = linear

first difference = same

Fig. 4 Typical graphical solution.



Student Work Sample H

③ Sally is not correct because only 38 people can sit around the table not 39.

rate of change

initial value

④ equation - $y = mx + b$

what it goes up by

$p = 3n + 2$ ← what it starts at

(people)

⑤ The pattern rule is going up by 3.

⑥ - First Differnce
→ because it goes up by 3. It is also in a Streight line

- Linear or non linear
→ linear, because the graph goes up by the same number and the points are in a Streight line.

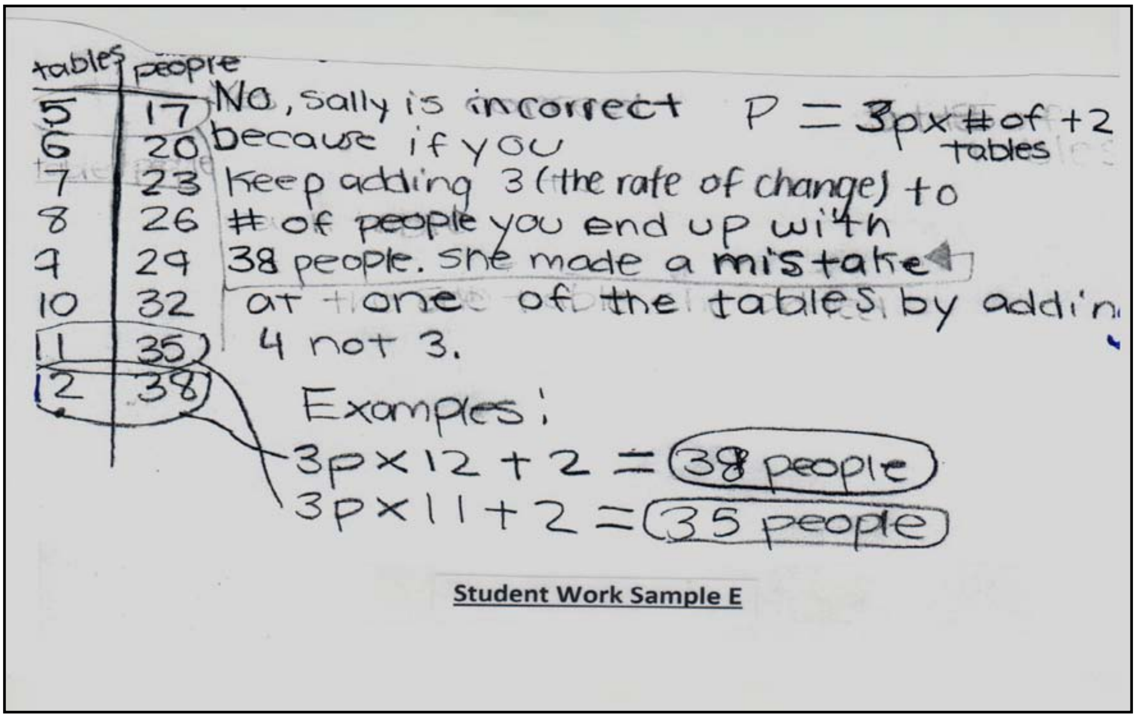
⑦ - Direct or Partial variation
→ Partial variation, because the graph and table don't start at 0, 0.

⑧ $p = 3n + 2$ $n = 12$ - In conclusion, Sally is wrong because only 38 people can sit.

$p = 3(12) + 2$

$p = 38$

Fig. 5 Student work sample H.



tables	people
5	17
6	20
7	23
8	26
9	29
10	32
11	35
12	38

No, sally is incorrect because if you Keep adding 3 (the rate of change) to # of people you end up with 38 people. She made a mistake at one of the tables by addin 4 not 3.

$P = 3p \times \# \text{ of tables} + 2$

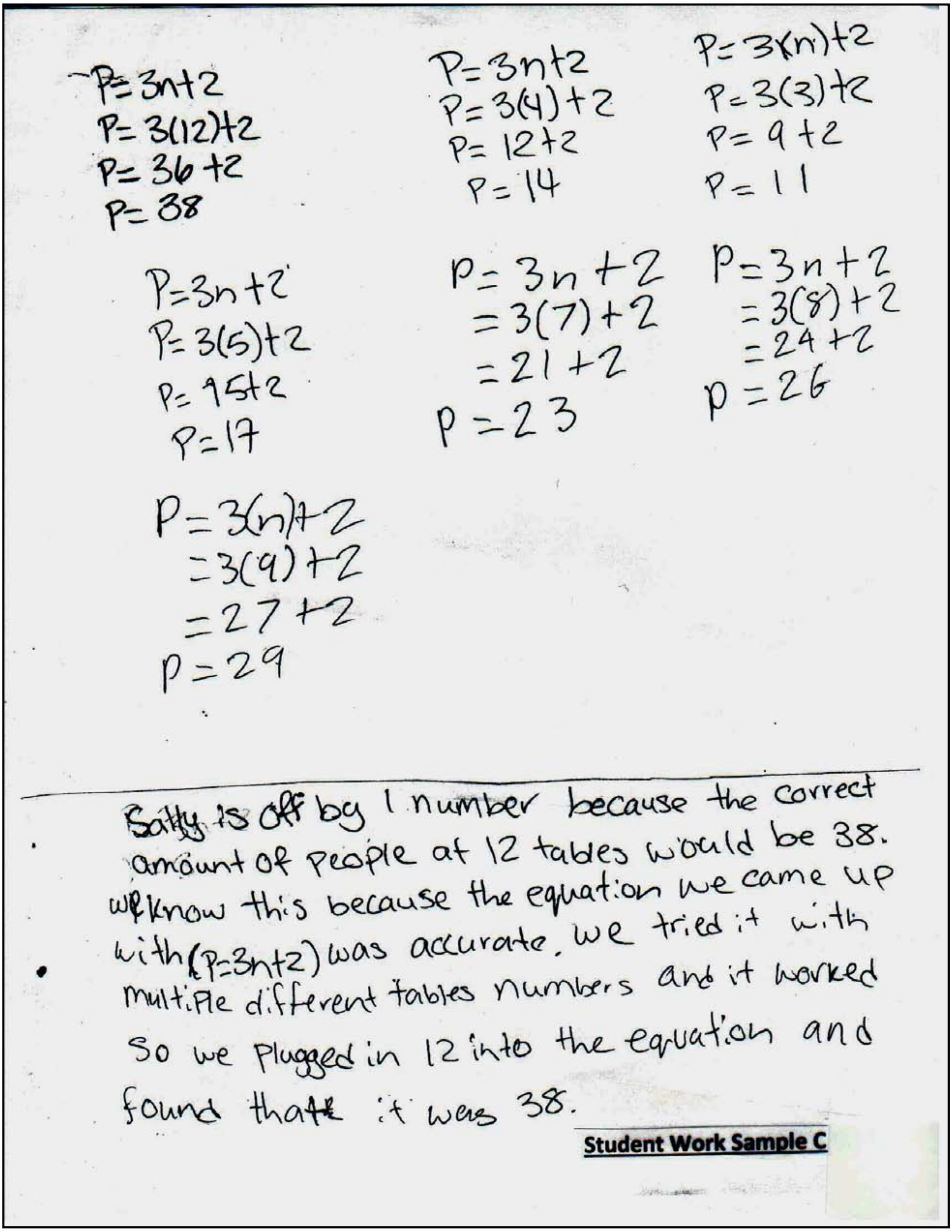
Examples:

$3p \times 12 + 2 = \boxed{38 \text{ people}}$

$3p \times 11 + 2 = \boxed{35 \text{ people}}$

Student Work Sample E

Fig. 6 Student work sample E.



$P=3n+2$
$P=3(12)+2$
$P=36+2$
$P=38$

$P=3n+2$
$P=3(4)+2$
$P=12+2$
$P=14$

$P=3(n)+2$
$P=3(3)+2$
$P=9+2$
$P=11$

$P=3n+2$
$P=3(5)+2$
$P=15+2$
$P=17$

$p=3n+2$
$=3(7)+2$
$=21+2$
$p=23$

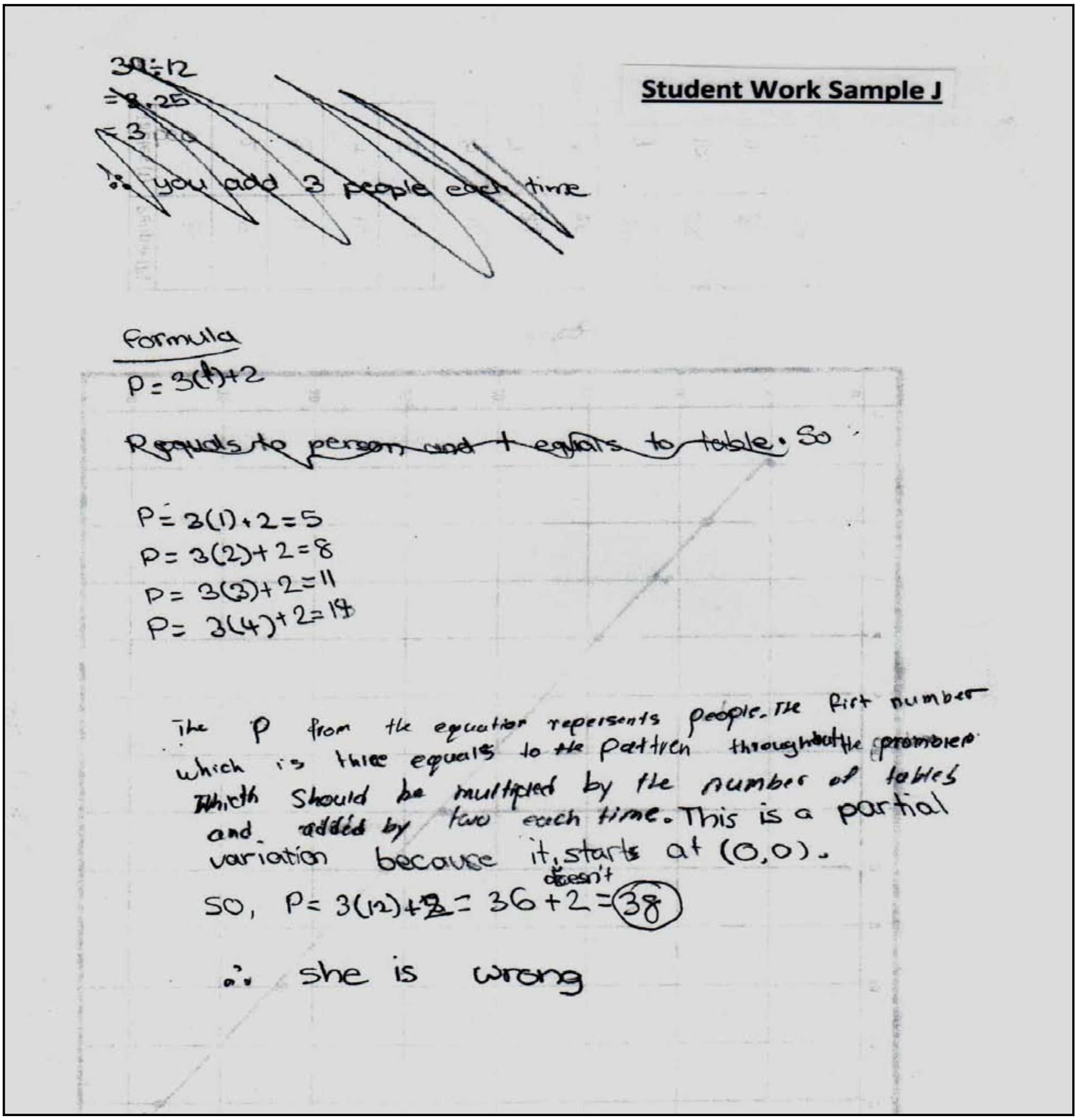
$P=3n+2$
$=3(8)+2$
$=24+2$
$p=26$

$P=3(n)+2$
$=3(9)+2$
$=27+2$
$p=29$

Sally is off by 1 number because the correct amount of people at 12 tables would be 38. we know this because the equation we came up with(P=3n+2) was accurate. we tried it with multiple different tables numbers and it worked so we plugged in 12 into the equation and found that it was 38.

Student Work Sample C

Fig. 7 Student work sample C.



Student Work Sample J

~~$39 \div 12$~~
~~$= 3.25$~~
~~$= 3$~~
~~∴ you add 3 people each time~~

Formula

$P = 3(t)+2$

~~Requals to person and t equals to table~~. So

$P = 3(1)+2=5$
$P = 3(2)+2=8$
$P = 3(3)+2=11$
$P = 3(4)+2=14$

The P from the equation repersents people. The firt number which is three equals to the pattren throughout the promblem Which should be multipled by the number of tables and added by two each time. This is a partial variation because it doesn't starts at (0,0).

So, $P = 3(12)+2 = 36+2 = \boxed{38}$

∴ she is wrong

Fig. 8 Student work sample J.

8. Conjectures and Predictions

When the whole class discussion began, the teachers posed these questions: "How would your solution change if two people were able to sit at each end of the trapezoid?" Students were able to quickly identify that this modification affected only the intercept. For the questions "How would your solution change if the tables allowed three people on the long side and two people on the parallel shorter side of each trapezoid? What parts of your solution does this affect?" students were able to quickly identify that these changes affected only the slope of the line, and produced a modified equation without resorting to any computations or creating any diagrams. Students were then asked to pose similar questions to their peers, with correct results quickly offered.

The teachers then posed this question: “What would change if the tables were not trapezoids, but some other geometrical shape?” Rectangles were quickly identified as simpler cases of the trapezoid problem, and solutions were offered to various rectangle scenarios. Students were able to generalize an equation for rectangles with any number of people on each side. However, most students were unable to make a similar generalization for the case of trapezoidal tables. This may have been due to having to consider three different possibilities for the number of people on each side of the trapezoid (long side, short side, ends).

Equilateral triangles were the first situation in which some students resorted to drawing diagrams before offering solutions. Isosceles triangles led to students posing the question “What would be the impact of changing the configuration?” For example, one student posed the following scenario. “What would happen if the tables are isosceles triangles, where one person can sit at the short side and two people at each of the long sides?” This led to students discussing possible configurations of the tables, rather than simply pushing the short sides together. The discussion then turned to other configurations, such as “Rather than one long row, what about two rows of trapezoidal tables pushed together? What happens if the trapezoidal tables are combined into blocks of four tables each?” At this stage, most students resorted to drawing diagrams before offering solutions. The discussion naturally led to questions about possible configurations of hexagonal tables, circular tables, and even mixed configurations including several table shapes in combination.

In one class, a discussion occurred when dealing with circular tables. There was no disputing that when circular tables were pushed together, each table lost one seat at the point of tangency. However, in considering the physical situation, one student pointed out that the tables might lose more than one seat, since positions near the point of tangency might not have sufficient room for people to be seated due to the circular shape of the table. A second student conjectured that this situation might be dependent on the diameters of the tables. When the diameters were fairly small, more than one seat would be lost. When diameters were larger, only the seats at the point of tangency would be lost. Since no actual circular tables were available in the classroom, the conjecture could not be tested. However, several students decided to go to the library after class to test the conjecture, since the library tables were circular. Since all the library tables were the same diameter, only part of the conjecture could be tested.

Teachers reported that students were able to identify the impact of some parameter changes; for example, changes to slope or intercept in graphical and algebraic solutions. Consequences for other changes (e.g., altering table configurations) would sometimes require additional investigation. The teachers believed that these “What If?” questions pushed students to engage in higher order thinking. Both the teachers felt that the task was valuable to push students to extend their thinking beyond the basic model. The teachers also reported that students remained engaged throughout the class, and some were reluctant to leave the class when the bell rang.

9. Discussion and Limitations

Based on the student work samples, the trapezoidal tables task addresses all the principles for an MEA [1-2]. Table 1 provides a summary of how this task qualifies as an MEA. One element of modeling that was absent was the “try and revise” procedure common to model building in the real world [2]. This was a consequence of the simplest prototype nature of the problem posed, such that a model can be formulated that solves the problem without the need for revisions and retesting. However, the extended “What If?” discussion did result in potential solutions for more intricate modeling situations.

Table 1 The trapezoidal tables task as an MEA.

MEA Criteria	The Trapezoidal Tables Task
Personal Meaningfulness	The task is clearly related to the real world. Students could envision arranging tables to seat a specified number of people.
Model Construction	The task influences students through the use of introductory diagrams. However, the statement of the task makes construction of a mathematical model an obvious solution technique.
Self-Evaluation	By positing a question to be answered (can Sally seat 39 people with 12 tables?), students are given not only a motivation to model, but a reason to evaluate the results of the modeling process
Model Generalizability	While not explicit in addressing this principle, the resulting model could clearly be generalized to respond to other similar questions. This principle encourages students to recognize structural similarities in analogous situations, one of the Mathematical Practices of the Common Core Standards (United States Department of Education, 2010). This dimension is explored further in the discussion of “What If?” questions.
Model Documentation	This was explicitly requested in the task statement. However, it was clear from the student work samples that written explanations and justification were an expectation in the participant classes. However, this was also one of the most problematic aspects of the student work samples, as discussed previously.
Simplest Prototype	This was a quite basic modeling situation.
*Links to prior knowledge	Students had knowledge of linear relations, first differences, graphing, algebraic representations, modeling.
*Requires higher level thinking	The What If? questions addressed higher order thinking skills.

Note. * see references [1-2].

No explicit discussion or definition of HOTS occurred with the teachers prior to this activity. Thus, the teachers involved did not use a taxonomy such as Marzano’s New Taxonomy to identify higher-order thinking. However, both teachers indicated that they recognized higher order thinking when it was expressed by students during the “What If?” discussion. The discussion clearly involved elements of Marzano’s taxonomy that are considered higher-order thinking, including generalizing, specifying (predicting), and formulating hypotheses. Although the students exhibited many higher-order thinking skills in the whole class discussion, training teachers in the use of Marzano’s taxonomy may have increased the quality of the discourse.

Another limitation was the timing of this activity. Being presented late in the semester clearly influenced how students treated the problem, many of them appeared to treat the problem as an application of content and concepts already learned in class, including partial variation, which was not necessary to solve the trapezoidal tables modeling problem. We recommend that this (or other MEAs) be given near the beginning of the semester, prior to the students being exposed to much of the content necessary for the model.

While our study involved Grade 9 students, this task can be used in any middle school grade. In the earlier grades, we would expect less reliance on algebraic solutions and more models involving manipulatives, graphs, or tables. It is also useful to present MEAs in the later grades. These model eliciting activities can be more fulsome, and involve higher level mathematical concepts and solutions, as well as more sophisticated justifications. However, the power of conjecture and prediction must still be a major component of such tasks.

10. Conclusion

Mathematical modeling is an important dimension of students' mathematical development, and a significant goal of mathematics education. By considering probable student behaviors when confronted with a task, and identifying desired behaviors, teachers can identify MEAs that will enrich students' mathematical development; "... modeling can be seen as a means of recognizing the potential of mathematics as a critical tool to interpret and understand reality, the communities children live in, or society in general" [7, p. 20].

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