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An Exact Solution of Steady Plane Rotating Aligned MHD Flows using Martin's Method in Magnetograph Plane

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Abstract: A steady plane MHD flows of an incompressible rotating viscous fluid with infinite electrical conductivity have been considered. Following Martin's approach results from differential geometry have been employed to recast the basic equations in (ϕ, ψ) coordinates. Further a new coordinate system (ϕ, y) is used to recast the flow equations in the new form. Lastly, an exact solution is obtained for parabolic magnetic field lines

Keywords: Martin's method, exact solution, stream function, aligned MHD flow, rotating frame.

1. Introduction

The theory of rotating fluid is highly important because of its occurrence in various natural phenomena and for its applications in various technological solutions which are directly governed by the actions of Coriolis force. The broad subject of oceanography, meteorology, atmospheric science and limnology all contain some important and essential features of rotating fluids. The current interest in the study of magnetohydrodynamics of rotating fluids has been motivated by several important problems like maintenance and secular variations of the earth's magnetic field; the internal rotation rate of the sun, the structure of rotating magnetic stars, the planetary and solar dynamo problems and centrifugal machines etc. These studies have been made partly in an effect to provide answers to these problems and partly to gain understanding of fluids behavior for various conditions and configurations.

Transformation techniques are often employed for solving non-linear partial differential equations. Navier-Stokes equations are non-linear partial differential equations. For this reason there exists only a limited number of exact solution in which the non-linear terms do not disappear automatically. Exact solutions are very important not only because they are solutions of some fundamental flows but also because they serve as accuracy checks for experimental, numerical and asymptotic methods. Martin [1] introduced a new method to study two dimensional steady plane flows of viscous fluid in the absence of external forces. In this approach a curvilinear coordinate system (ϕ, ψ) in the plane of flow is introduced and the coordinate lines $\psi = \text{constant}$ are taken as streamlines of the flow while the coordinate lines $\phi = \text{constant}$ are, at first, left arbitrary. In the first approach of Martin [1] the metric are taken as unknown functions. As a result the coefficients E, F and G of the first fundamental form of the metric are taken as unknown functions. As a result partial differential equations in terms of E, F and G, the vorticity etc. are obtained as functions of ϕ and ψ. In the second approach, the quantities E, F and G are again introduced but are eliminated in favour of the vorticity ω and energy function h, which then become the

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unknown functions of ϕ, ψ. Chandna and Garg [2], Chandna, Barron and Garg [3], Bagewadi and Siddabasappa [4] applied Martin's approach for MHD flow problems and obtained solutions. Naeem [5] used Martin's method to study Navier-Stokes equations for steady plane viscous incompressible fluids and determined some exact solutions. Chandna and Labropulu [6] used von Mises coordinates (x, ψ) that require the use of $\phi = x$ in Martin's coordinates (ϕ, ψ) to find exact solutions of steady plane viscous magnetohydrodynamic flows. Naeem and Nadeem [7] extended Martin's approach to study steady plane flows of an incompressible fluid of variable viscosity for an arbitrary state equation. Naeem and Ali [8] used Martin's approach to equations governing the steady plane flows of an incompressible fluid of variable viscosity by taking $\phi = x$. Labropulu and Chandna [9] extended Martin's approach to study steady plane MHD aligned flows. Ali, Ara and Khan [10], Thakur, Manoj Kumar and Mahan [11], Manoj Kumar, Thakur, Singh and Mahan [12] and other authors applied Martin's approach for MHD flow problems and obtained exact solutions. Naeem, Mansoor, Khan and Aurangzaib [13] extended the approach of Labropulu and Chandna [9] to study plane flows of an incompressible fluid of variable viscosity for arbitrary state equation and presented some exact solutions. Using the same approach again Naeem, Mansoor, Khan and Aurangjaib [14] found out exact solutions of steady plane flows of an incompressible fluid of variable viscosity in the presence of unknown external force. Also Bagewadi and Siddabassapa [15], Singh, Singh and Ram Babu [16], Singh and Singh [17], Thakur and Manoj Kumar [18], Sayantan Sil and Manoj Kumar [19], Sayantan Sil and Manoj Kumar [20] and some other authors have studied rotating MHD flows and have found out exact solutions.

In this paper we consider steady plane aligned MHD flows of an incompressible rotating viscous fluid with infinite electrical conductivity. Following Martin's approach we have employed the results of differential geometry to transform the basic equations in (ϕ, ψ) coordinates. We further use a new coordinate system (ϕ, y) to recast the flow equations in new form and have obtained an exact solution for parabolic magnetic field lines.

2. Basic Equations

The steady MHD flow in a rotating frame of a homogenous, incompressible, viscous fluid with infinite electrical conductivity is given by the following system of equations:

$$\nabla \cdot \mathbf{V} = 0, \qquad \text{(Continuity)} \qquad (1)$$

$$\rho\left[\left(\mathbf{V}\cdot\nabla\right)\mathbf{V} + 2\mathbf{\Omega}\times\mathbf{V} + \mathbf{\Omega}\times\left(\mathbf{\Omega}\times\mathbf{r}\right)\right] + \nabla P =$$

$$\eta\nabla^2\mathbf{V} + \mu\left(\nabla\times\mathbf{H}\right)\times\mathbf{H} \qquad \text{(Linear Momentum)} \qquad (2)$$

$$\nabla\times\left(\mathbf{V}\times\mathbf{H}\right) = \mathbf{0}, \qquad \text{(Diffusion)} \qquad (3)$$

$$\nabla\cdot\mathbf{H} = 0. \qquad \text{(Solenoidal)} \qquad (4)$$

In the above equations $\mathbf{V}$= velocity vector, $\mathbf{r}$ = radius vector, $\mathbf{H}$ = magnetic field intensity, P = pressure, $\rho =$ fluid density, $\mathbf{\Omega} =$ angular velocity vector, $\mu =$ magnetic permeability, $\eta =$ constant viscosity.

We consider the flow to be two-dimensional so that **V** and **H** lie in a plane defined by rectangular co-ordinates (x, y) and all the flow variables are functions of x and y. We introduce the following functions:

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \qquad \text{(Vorticity Function)} \qquad (5)$$

$$Q = \frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}, \qquad \text{(Current Density Function)} \qquad (6)$$

$$B^* = \frac{1}{2}\rho V^2 + P' + \frac{1}{2}\rho\left|\mathbf{\Omega}\times\mathbf{r}\right|^2, \text{(Bernoulli Function)} \qquad (7)$$

where $V^2 = u^2 + v^2$ and P' is the reduced pressure, $P' = P - \frac{1}{2}\rho\left|\mathbf{\Omega}\times\mathbf{r}\right|^2$ and last term being the centrifugal contribution of the pressure, u, v are the components of the velocity field $\mathbf{V}$, H_1 and H_2 are the components of magnetic field vector $\mathbf{H}$.

Now the system of equations (1)-(4) is replaced by the following system:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{8}$$

$$\frac{\partial B^*}{\partial x}+\eta\frac{\partial \omega}{\partial y}-2\rho\Omega v-\rho v\omega+\mu H_2 Q=0, \tag{9}$$

$$\frac{\partial B^*}{\partial y}-\eta\frac{\partial \omega}{\partial x}+2\rho\Omega u+\rho u\omega-\mu H_1 Q=0, \tag{10}$$

$$uH_2-vH_1=C, \tag{11}$$

$$\frac{\partial H_1}{\partial x}+\frac{\partial H_2}{\partial y}=0, \tag{12}$$

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\omega, \tag{13}$$

$$\frac{\partial H_2}{\partial x}-\frac{\partial H_1}{\partial y}=Q, \tag{14}$$

where C is an arbitrary constant of integration obtained from the diffusion equation (3). The advantage of this system over the original system is that the order of partial differential equations has decreased from two to one.

We now consider aligned flow in which velocity field is everywhere parallel to magnetic field, so that

$$\mathbf{V}=\beta\mathbf{H} \text{ i.e., } u=\beta H_1 \text{ and } v=\beta H_2, \tag{15}$$

where β is some unknown scalar function such that

$$\mathbf{H}\cdot\nabla\beta=0, \tag{16}$$

is the condition satisfied by β as obtained from equation (1), (4) and (15). Using (15) in the above system of equations (8) to (14), we get the following system of partial differential equations:

$$H_1\frac{\partial \beta}{\partial x}+H_2\frac{\partial \beta}{\partial y}=0, \text{ (Continuity)} \tag{17}$$

$$\frac{\partial B^*}{\partial x}+\eta\frac{\partial \omega}{\partial y}-2\rho\Omega\beta H_2-\rho\omega\beta H_2+\mu H_1 Q=0, \text{ (Linear Momentum)} \tag{18}$$

$$\frac{\partial B^*}{\partial y}-\eta\frac{\partial \omega}{\partial x}+2\rho\Omega\beta H_1+\rho\omega\beta H_1-\mu H_1 Q=0, \text{ (Linear Momentum)} \tag{19}$$

$$\frac{\partial H_1}{\partial x}+\frac{\partial H_2}{\partial y}=0, \text{ (Solenoidal)} \tag{20}$$

$$\beta Q+H_2\frac{\partial \beta}{\partial x}-H_1\frac{\partial \beta}{\partial y}=\omega, \text{ (Vorticity)} \tag{21}$$

$$\frac{\partial H_2}{\partial x}-\frac{\partial H_1}{\partial y}=Q, \text{ (Current Density)} \tag{22}$$

these are the six partial differential equations in six unknown functions

$H_1(x,y)$, $H_2(x,y)$, $B^*(x,y)$, $\omega(x,y)$, $Q(x,y)$ and $\beta(x,y)$. Once a solution of this system is determined, the pressure function $P(x,y)$ and the velocity vector $\mathbf{V}$ can be obtained.

3. Some Results from Differential Geometry

Let the equations

$$x=x(\phi,\psi),\ y=y(\phi,\psi), \tag{23}$$

define a system of curvilinear coordinates in the (x,y)-plane. In the curvilinear coordinate system (ϕ,ψ) the squared element of the arc length is given by

$$ds^2=E\,d\phi^2+2F d\phi\, d\psi+G d\psi^2, \tag{24}$$

where

$$E=\left(\frac{\partial x}{\partial \phi}\right)^2+\left(\frac{\partial y}{\partial \phi}\right)^2,\ F=\left(\frac{\partial x}{\partial \phi}\right)\left(\frac{\partial x}{\partial \psi}\right)+\left(\frac{\partial y}{\partial \phi}\right)\left(\frac{\partial y}{\partial \psi}\right),$$

$$G=\left(\frac{\partial x}{\partial \psi}\right)^2+\left(\frac{\partial y}{\partial \psi}\right)^2. \tag{25}$$

Equation (23) can be used to obtain

$$\frac{\partial x}{\partial \phi}=J\frac{\partial \psi}{\partial y},\ \frac{\partial x}{\partial \psi}=-J\frac{\partial \phi}{\partial y},\ \frac{\partial y}{\partial \phi}=-J\frac{\partial \psi}{\partial x},\ \frac{\partial y}{\partial \psi}=J\frac{\partial \phi}{\partial x}, \tag{26}$$

provided $0<|J|<\infty$, where J denotes the Jacobian.

$$J=\left(\frac{\partial x}{\partial \phi}\right)\left(\frac{\partial y}{\partial \psi}\right)-\left(\frac{\partial x}{\partial \psi}\right)\left(\frac{\partial y}{\partial \phi}\right)=\pm\sqrt{EG-F^2}=\pm W. \tag{27}$$

Let α be the angle made by the tangent to the coordinate line ϕ = constant directed in the sense of increasing ψ, with the x-axis, we have from differential geometry following Nath and Chandna [21].

$$\frac{\partial x}{\partial \psi} = \sqrt{G}\cos\alpha, \frac{\partial y}{\partial \psi} = \sqrt{G}\sin\alpha ,$$

$$\frac{\partial x}{\partial \phi} = \frac{F}{\sqrt{G}}\cos\alpha + \frac{J}{\sqrt{G}}\sin\alpha ,$$

$$\frac{\partial y}{\partial \phi} = \frac{F}{\sqrt{G}}\sin\alpha - \frac{J}{\sqrt{G}}\cos\alpha ,$$

$$\frac{\partial \alpha}{\partial \phi} = \left(\frac{J}{G}\right)\Gamma_{12}^2 , \frac{\partial \alpha}{\partial \psi} = \left(\frac{J}{G}\right)\Gamma_{11}^2 ,$$

and $$K = \frac{1}{W}\left[\frac{\partial}{\partial \psi}\left\{\frac{J}{G}\Gamma_{12}^2\right\} - \frac{\partial}{\partial \phi}\left\{\frac{J}{G}\Gamma_{11}^2\right\}\right] = 0 , \qquad (28)$$

where

$$\Gamma_{11}^2 = \frac{1}{2W^2}\left[F\frac{\partial G}{\partial \psi} - 2G\frac{\partial F}{\partial \psi} + G\frac{\partial G}{\partial \phi}\right] ,$$

$$\Gamma_{12}^2 = \frac{1}{2W^2}\left[F\frac{\partial G}{\partial \phi} - G\frac{\partial F}{\partial \psi}\right]$$

and K is the Gaussian curvature.

4. New Form of Flow Equations

The solenoidal equation (20) implies the existence of magnetic flux function $\phi = \phi(x, y)$ such that

$$\frac{\partial \phi}{\partial x} = -H_2 , \frac{\partial \phi}{\partial y} = H_1 . \qquad (29)$$

4.1 Linear Momentum Equations

Employing equation (29) in equations (18) and (19) and making use of (26), we find these equations in (ϕ, ψ) coordinates as follows:

$$\frac{\partial B^*}{\partial \phi}\frac{\partial y}{\partial \psi} - \frac{\partial B^*}{\partial \psi}\frac{\partial y}{\partial \phi} + \eta\left(-\frac{\partial \omega}{\partial \phi}\frac{\partial x}{\partial \psi} + \frac{\partial \omega}{\partial \psi}\frac{\partial x}{\partial \phi}\right)$$
$$-\left[(\omega + 2\Omega)\rho\beta - \mu Q\right]\frac{\partial y}{\partial \phi} = 0 , \qquad (30)$$

$$\frac{\partial B^*}{\partial \phi}\frac{\partial x}{\partial \psi} - \frac{\partial B^*}{\partial \psi}\frac{\partial x}{\partial \phi} + \eta\left(\frac{\partial B^*}{\partial \phi}\frac{\partial y}{\partial \psi} - \frac{\partial \omega}{\partial \psi}\frac{\partial y}{\partial \phi}\right)$$
$$-\left[(\omega + 2\Omega)\rho\beta - \mu Q\right]\frac{\partial x}{\partial \phi} = 0. \qquad (31)$$

Multiplying equation (30) by $\frac{\partial x}{\partial \phi}$ and (31) by $\frac{\partial y}{\partial \phi}$ and subtracting, we get

$$J\frac{\partial B^*}{\partial \phi} = \eta\left[F\frac{\partial \omega}{\partial \phi} - E\frac{\partial \omega}{\partial \psi}\right], \qquad (32)$$

again multiplying equation (30) by $\frac{\partial x}{\partial \psi}$ and (31) by $\frac{\partial y}{\partial \psi}$ and subtracting, we get

$$J\frac{\partial B^*}{\partial \psi} = \eta\left[-F\frac{\partial \omega}{\partial \psi} + G\frac{\partial \omega}{\partial \phi}\right] - J\left[(\omega + 2\Omega)\rho\beta - \mu Q\right]. \qquad (33)$$

4.2 Equation of Continuity

Using equation (29) in the equation of continuity (17) and transforming to (ϕ, ψ)-net, we have

$$\frac{\partial \phi}{\partial y}\left[\frac{\partial \beta}{\partial \phi}\frac{\partial \phi}{\partial x} + \frac{\partial \beta}{\partial \psi}\frac{\partial \psi}{\partial x}\right] - \frac{\partial \phi}{\partial x}\left[\frac{\partial \beta}{\partial \phi}\frac{\partial \phi}{\partial y} + \frac{\partial \beta}{\partial \psi}\frac{\partial \psi}{\partial y}\right] = 0 , \qquad (34)$$

which on simplification gives

$$\frac{\partial \beta}{\partial \psi} = 0 . \qquad (35)$$

4.3 Equation of Vorticity

Employing (29) in the vorticity equation (21), we get

$$\omega = \beta Q - \frac{\partial \beta}{\partial x}\frac{\partial \phi}{\partial x} - \frac{\partial \beta}{\partial y}\frac{\partial \phi}{\partial y} , \qquad (36)$$

equation (36) together with (25) , (26) and (35) gives

$$\beta Q - \frac{G}{J^2}\frac{\partial \beta}{\partial \phi} = \omega . \qquad (37)$$

4.4 Solenoidal and Current Density Equation

Nath and Chandna [21] have proven that the solenoidal condition yields

$$WH = \sqrt{G} , H_1 + iH_2 = \left(\frac{\sqrt{G}}{W}\right)\exp(i\alpha) , \qquad (38)$$

where $H=\left(H_1^2+H_2^2\right)^{1/2}$ and α is the angle between the tangent to the coordinate line $\phi=$ constant, directed in the sense of increasing ψ and the x-axis.

Following Nath and Chandna [21], the new form of current density equation is given by

$$Q=\frac{1}{W}\left[\frac{\partial}{\partial\phi}\left(\frac{G}{W}\right)-\frac{\partial}{\partial\psi}\left(\frac{F}{W}\right)\right] \quad . \qquad (39)$$

5. Exact Solution

Let $\quad x=f(y)+g(y)\Upsilon(\phi)$, (40)

where $\Upsilon(\phi)$ is an unknown function satisfying $\Upsilon'(\phi)\neq 0$, be the family of magnetic lines for the given flow problem.

Now employing the coordinate $(\phi,\psi=y)$ and equation (40) in equation (25) and (27), we have

$$E=g^2(y)\,\Upsilon'^2(\phi) \quad , \quad G=1+\left\{f'(y)+g'(y)\Upsilon(\phi)\right\}^2$$

$$F=\left\{g(y)f'(y)+g(y)g'(y)\,\Upsilon(\phi)\right\}\Upsilon'(\phi),$$

$$J=W=g(y)\,\Upsilon'(\phi) \qquad (41)$$

Now we take $f(y)=m_1y^2+m_2y$ and $g(y)=1$ then (40) becomes

$$x=m_1y^2+m_2y+\Upsilon(\phi)\ ,\ \ \Upsilon'(\phi)\neq 0\ . \qquad (42)$$

Using (42) in (41), we obtain

$$E=\Upsilon'^2(\phi),\ G=(2m_1y+m_2)^2+1$$

$$F=(2m_1y+m_2)\Upsilon'(\phi)\ ,\quad J=\Upsilon'(\phi)\ . \qquad (43)$$

Now, we write the equations (32), (33), (37) and (39) in coordinates (ϕ,ψ) by using the equation (43), we have

$$\frac{\partial B^*}{\partial\phi}=\eta\left[\left(2m_1y+m_2\right)\frac{\partial\omega}{\partial\phi}-\Upsilon'(\phi)\left(\frac{\partial\omega}{\partial y}\right)\right], \qquad (44)$$

$$\frac{\partial B^*}{\partial\phi}=\frac{\eta}{\Upsilon'(\phi)}\left[\begin{array}{l}\left(2m_1y+m_2\right)^2\dfrac{\partial\omega}{\partial\phi}+\dfrac{\partial\omega}{\partial\phi}\\ -\left(2m_1y+m_2\right)\Upsilon'(\phi)\left(\dfrac{\partial\omega}{\partial y}\right)\end{array}\right]$$

$$-\Upsilon'(\phi)\left[(\omega+2\Omega)\rho\beta(\phi)-\mu Q\right] \ , \qquad (45)$$

$$\omega=\beta(\phi)\,Q-\left[\frac{1+\left(2m_1y+m_2\right)^2}{\Upsilon'^2(\phi)}\right]\beta'(\phi)\ , \qquad (46)$$

$$Q=\frac{1}{\Upsilon'^3(\phi)}\left[\begin{array}{l}2m_1\Upsilon'^2(\phi)+\Upsilon''(\phi)\\ +\left(2m_1y+m_2\right)^2\Upsilon''(\phi)\end{array}\right]. \qquad (47)$$

By integrability condition $\dfrac{\partial^2B^*}{\partial\phi\partial y}=\dfrac{\partial^2B^*}{\partial y\partial\phi}$, eliminating ω, Q from equation (44) and (45) with the help of equation (46) and (47), we find

$$\sum a_n(\phi)\left[(2m_1y+m_2)\right]^n=0 \quad , \qquad (48)$$

where,

$$a_0(\phi)=a_4(\phi)-4m_1\left[\frac{\Upsilon''(\phi)}{\Upsilon'^3(\phi)}\right]'+12m_1^2\frac{\Upsilon''(\phi)}{\Upsilon'^2(\phi)},$$

$$a_1(\phi)=-\frac{4m_1}{\eta}\left[\begin{array}{l}\left\{\rho\beta^2(\phi)-\mu\right\}\dfrac{\Upsilon''(\phi)}{\Upsilon'^3(\phi)}\\ +\left\{\dfrac{\beta(\phi)}{\Upsilon'^2(\phi)}+2\Omega\right\}\rho\beta'(\phi)\end{array}\right] \ ,$$

$$a_2(\phi)=2a_4(\phi)-12m_1\left[\frac{\Upsilon''(\phi)}{\Upsilon'^3(\phi)}\right] \ ,$$

$$a_3(\phi)=0$$

and

$$a_4(\phi)=\left[\frac{1}{\Upsilon'(\phi)}\left\{\frac{\Upsilon''(\phi)}{\Upsilon'^3(\phi)}\right\}'\right]'.$$

Since ϕ, y are independent variables, the identity (5.48) can only satisfy if $a_0(\phi)$, $a_1(\phi), a_2(\phi)$ and $a_4(\phi)$ vanish identically. Using consequences $a_4(\phi)=0$, $a_2(\phi)=0$ in $a_0(\phi)=0$ and $a_1(\phi)=0$, we have

$$\Upsilon(\phi)=c_1\phi+c_2 \ , \ \beta(\phi)=\beta_0 \ , \qquad (49)$$

where $c_1 \neq 0$, c_2 and $\beta_0 \neq 0$ are arbitrary constants.

From (49) and (42), we obtain

$$c_1\phi(x,y)+c_2 = x-m_1y^2-m_2y \ , \qquad (50)$$

for the chosen parabolic magnetic field lines.

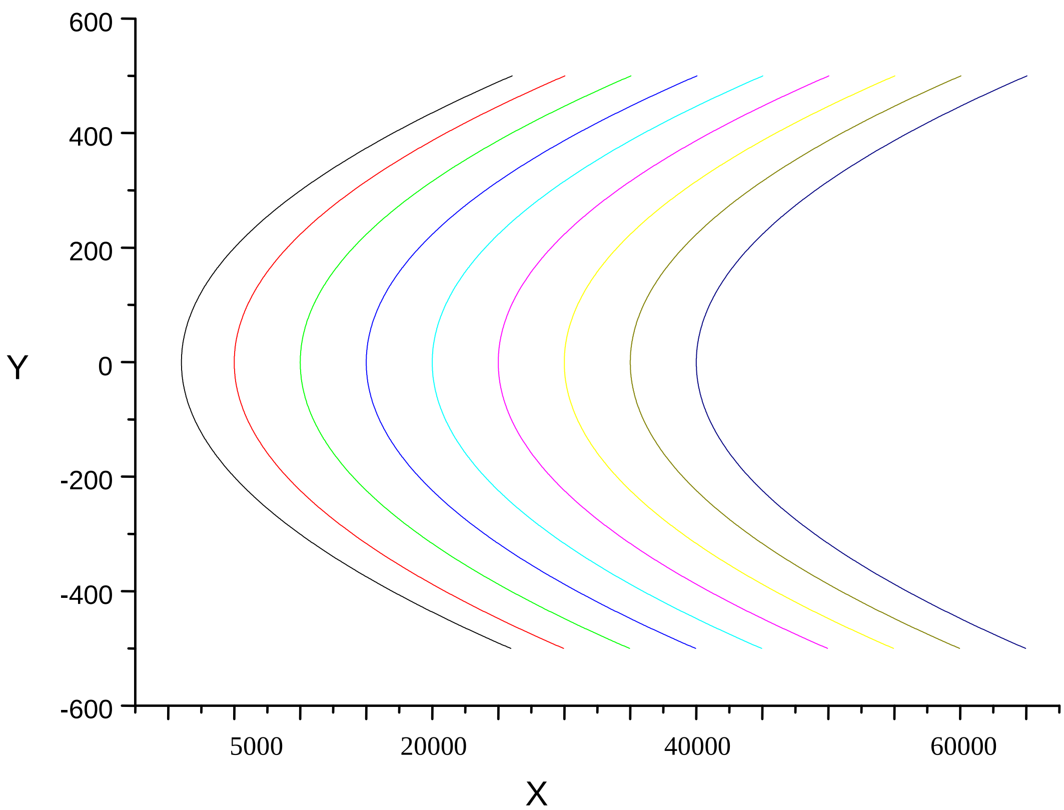

Fig. 1 Parabolic magnetic field lines.

Using (49) and (50) in (29), (15), (5), (6), (7) and (17)-(22), we find

$$H_1 = -\left(\frac{2m_1y+m_2}{c_1}\right), \qquad (51)$$

$$H_2 = -\frac{1}{c_1} \ , \qquad (52)$$

$$u = -\beta_0\left(\frac{2m_1y+m_2}{c_1}\right) \ , \qquad (53)$$

$$v = -\frac{\beta_0}{c_1}, \qquad (54)$$

$$P = \frac{1}{c_1}\left[\frac{2\mu m_1}{c_1}-\left(\frac{2m_1\beta_0}{c_1}+2\Omega\right)\rho\beta_0\right]x + 2\left[\frac{\rho\beta_0\Omega}{c_1}-\frac{\mu m_1}{c_1^2}\right]\left(m_1y^2+m_2y\right) - \frac{\rho\beta_0^2\left(1+m_2^2\right)}{2c_1^2}+P_0 \ , \qquad (55)$$

$$\omega = \frac{2m_1}{c_1}\beta_0 \ , \qquad (56)$$

and

$$Q = \frac{2m_1}{c_1} \,, \tag{57}$$

where P_0 is an arbitrary constant.

In absence of rotating frame i.e. $\mathbf{\Omega} = \mathbf{0}$, we get the results of Manoj Kumar et al. [12] and O.P Chandna and F. Labropulu [6].

6. Conclusion

An approach for the determination of exact solution of steady plane viscous incompressible rotating fluid in magnetograph plane has been presented in the case of MHD parallel flows when electrical conductivity is infinite. The expressions for velocity, magnetic field and pressure distribution are found out. Parabolic magnetic field lines are also plotted. Also, the present analysis is more general and several results of various authors (as already mentioned in the text) can be recovered in the limiting cases.

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