Journal of Math Studies 4 (2017) 25-39
ISSN 2474-4808

Betty Jones & Sisters Publishing
http://www.bettyjonespub.com

A Whole-School Implementation of Math-Talk Learning Communities

Jeff Irvine
Brock University, Canada

Abstract: Math-Talk Learning Communities (MTLCs) are based on sociocultural views of learning, specifically social constructivism. MTLCs have been shown to benefit student learning across dimensions such as interpersonal skills, deep learning of mathematical concepts, growth in interpersonal skills, motivation, and engagement. After reviewing the benefits of MTLCs, this paper examines the successful implementation of a MTLC across a large elementary school in Ontario, Canada. Conditions critical for successful whole-school implementation are identified and possible replication of the MTLC in other locations is discussed.

Key words: Collective argumentation, exploratory talk, math talk learning community, three-part lesson

1. Introduction

Educators in this century have began to recognize the importance of schools focusing on educating the whole child. As Boaler (2008) notes, one of the goals of schools should be to produce citizens who treat each other with respect, who value the contributions of others with whom they interact, irrespective of their race, class or gender, and who act with a sense of justice in considering the needs of others in society [1, pp. 167-168].

Part of this focus therefore entails "teaching students appropriate interactions and "learning practices" [1, p.171]. This orientation requires a departure from the traditional teacher initiation-student response-teacher evaluation/feedback (IRE) model [2-3] and an awareness of and support for student discussion, because as Kapellidi (2015) aptly observes, "classroom interaction is one domain that is clearly distinctive from everyday talk" [2, p. 453]. Student talk must be focused on moving the collective forward in their knowledge, understanding, and ability to enunciate the mathematical principles underlying the curriculum. This position is radically different than the experience of most teachers, and even teacher candidates.

Bofferding and Kemmerle's study of elementary teacher candidates found that only 12.7% believed that students should be actively engaged in solving learner-generated problems, and 28.4% believed that the role of the teacher is to transmit information [4]. Given such data, attempts to move classrooms towards communities of practice may "require a reconceptualization of student participation in the practices of the everyday classroom" [5, p. 118]. Such a reconceptualization certainly would be beneficial, as there is evidence that collaborative classroom student discussion increases both equity and student achievement [3]. Critical to this collaborative discussion is the concept of exploratory talk, which involves students "engaging critically and constructively with each others' ideas" [6, p. 8]. Gillies and Khan (2009) found that students who were taught to use exploratory talk "produced significantly more and better arguments, expressed their arguments in a more cogent and explicit way, and provided more links and support to sustain their opinions than their untrained peers" [6, p. 8], which together significantly impacts students' ability to solve problems.

Corresponding author: Jeff Irvine, Ph.D., Sessional Instructor. E-mail: jeffrey.irvine@brocku.ca.

2. Math-Talk Learning Communities

Math-Talk Learning Communities (MTLCs) are central to the concept of cooperative student discussion. Hufford-Accles, Fuson, and Gamoran Sherin (2004) define a MLTC as “a classroom community in which the teacher and students use discourse to support the mathematical learning of all participants,” and whose “primary goal ... is to understand and extend one’s own thinking as well as the thinking of others in the classroom” [7, p. 82]. A key part of Hufford -Accles et al.’s definition is MTLCs’ focus on all participants’ mathematical learning and the emphasis on classroom discourse being used to advance everyone’s learning. The benefits of classroom discussion include allowing “students to test ideas, to hear and incorporate the ideas of others, to consolidate their thinking by putting ideas into words, and hence, to build a deeper understanding of key concepts” [8, p. 111]. MTLCs lead to student collaboration, taking responsibility when things go wrong, persistence in the face of adversity, respect for the ideas and perspectives of others, taking responsibility for each other’s learning, student confidence, as well as practice in interpreting and justifying positions [1].

Hufford-Accles et al. [7] have developed a set of rubrics to identify current and desired progress toward a MTLC based on four dimensions: questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning. In questioning, the shift is from teacher as sole questioner to students and teachers alike as questioners. In a MTLC, students increasingly explain and articulate their mathematical ideas, and there is a shift from the teacher as the source of all ideas to students having an influence on the direction of lessons. Finally, students take further responsibility for their own learning and evaluation, as well as for others’ learning. Still, establishing a MTLC requires a major paradigm shift for most teachers, as they no longer are the source of all knowledge and arbitrator of all truth. Mathematical sense-making becomes the prime criteria for judging the veracity of claims and solutions.

2.1 Epistemology

MTLCs are based on a socio-constructivist foundation [7, 9-11] and Piaget’s theories that sought to conceptualize “personal learning through the metaphor of emergent biological forms, the structures of which are conditioned but never determined by their contexts—hence his use of terms such as “assimilation” and “accommodation” [12, pp. 411-412]. For Piaget, learning was an individual but not isolated activity in which “the individual knower was engaged in the unrelenting project of assembling a coherent interpretive system, constantly updating and revising explanations and expectations to account for new experiences” [12, p. 413]. Of particular importance to Piaget was the role of conflict and confusion that serve to transform thought [11]. Thus, one of the dual functions of a MTLC is both to cause conflict and provide socially acceptable methods for resolving that conflict. Bauersfeld [13], in arguing for “mathematizing as a social practice,” also recognizes the multiplicity of positions within a mathematics classroom—sometimes competing, but often complementary.

Social constructivism foregrounds social interactions as the drivers of learning. Based on Vygotsky’s notions of the interpersonal preceding the intrapersonal, cognition is diffuse, distributed, and collective [12, p. 414]. Gergen [14] points out that language plays a very significant role in social constructivism, as a way to communicate meaning, where meaning is also context dependent. Vygotsky’s work provides some informative concepts for teachers, such as the zone of proximal development (ZPD)—the difference between what a learner can accomplish with the assistance of knowledgeable others (teachers, parents, other students) and what the learner can accomplish unaided [15]. The processes inside the ZPD are typically called scaffolding [15] and consist not of telling the student an answer but rather asking questions, suggesting directions, directing students to other resources, and providing encouragement.

Sociocultural theories are related to Vygotsky's metaphor of shared labour. Davis and Sumara [12] identify classroom related facets of this theory, such as emphasis on group processing, and the justification of positions within disciplines. There is a relationship here to situated cognition, with its concern for the processes by which individuals enter into established communities of practice [12]. There are five major dimensions of student classroom discourse: patterns of discourse, levels of response, the roles of teachers and students, sociomathematical norms, and mathematical practices in the classroom [8]. MTLCs aim to develop classroom communities of practice [16] in which newcomers are assimilated into the community of practice through participation.

Teacher behaviours represent a significant hurdle in the development of a MTLC as a community of practice [17]. Teachers need to know and practice behaviours that support the development of a MTLC and develop appropriate student behaviours. As Brown [5] puts it, teaching needs to viewed as a partnership where regular opportunities are provided for students to use speech in collaborative activities with others, to adopt different roles within the learning process, to change the ways in which they relate to each other, and to see that participation in learning is for everyone regardless of age, perceived ability, socio-economic status, ethic (sic), or religious background. (pp. 116-117)

In a MTLC, "participants consider all members of the community to be constructing their own knowledge and reflecting on and discussing this knowledge" [7, p. 83].

2.2 Exploratory Talk and Collective Argumentation

MTLC requires students to engage in exploratory talk "in which partners engage critically but constructively with each other's ideas" [18, p. 180]. Within the scope of exploratory talk, "Statements and suggestions are sought and offered for joint consideration. These may be challenged and counter-challenged, but challenges are justified and alternative hypotheses are offered. In exploratory talk, knowledge is made publicly accountable and reasoning is visible in the talk" [18]. A critical aspect of exploratory talk is that it is student-centred, with students working collaboratively to share and critique ideas independently of the teacher [10]. Wegeril and Mercer [18] have identified a number of norms for exploratory talk: all relevant information is shared, the group seeks to reach agreement, the group takes responsibility for decisions, reasons are expected, challenges are acceptable, alternatives are discussed before a decision is taken, and everyone in the group is encouraged to speak by other group members (p. 181).

Collective argumentation is closely related to exploratory talk. Brown and Renshaw [19] identify five principles underlying collective argumentation. Generalizability means that students communicate their ideas in a form that others can analyze critically. The objectivity principle states that ideas can only be rejected based on logical reasoning or past experience. Consistency requires that contradictory or mutually exclusive positions must be resolved through group logical argument. Consensus requires that all group members contribute to co-construction of a solution strategy. Finally, recontextualization involves presenting the co-constructed argument to the other group members for validation. By employing this set of principles, collective argumentation not only allows for the teaching of social norms but also places students in the role of mathematical investigators. Thus, students are involved in mathematical communities of practice [5]—that is, MTLCs.

2.3 Norms and Social Skills

A pre-requisite for a successful MTLC is the establishment of norms, and the teaching of social skills. If students are to participate appropriately in exploratory talk and collective argumentation, they need to have the social skills, and they need to be aware of and conform to collective norms. The primary goal of establishing norms is to create a safe space in the classroom so that students feel free to participate and take risks [20]. The act of teaching appropriate behaviours will encourage some students to participate more fully. Teaching students how to respond to other students' arguments, critique ideas, and resolve conflicts must be done early in the school year [11]. Establishing explicit expectations around behaviour increases student comfort levels [21] and raises the level of risk and challenge that students are willing to accept [20]. Naylor and Cowie [22] point out that peer behaviour is crucial; it includes constructive criticism, encourages questions, and builds knowledge and understanding. Thus, establishing appropriate norms for peer interactions supports a MTLC. Brown [5] proposes a negotiated class charter of values, to "reflect social virtues of engagement, courage, humility, honesty, restraint, persistence, and affirmation" (p. 120).

2.4 The Role of the Teacher in a MTLC

While the teacher's role is pivotal to the successful creation and function of the MTLC [8-9], the teacher's function is significantly different than the traditional teacher role of primary questioner and the purveyor of knowledge. The overarching function of the teacher is to teach students to think critically [23] and to cultivate a classroom culture in which mathematical knowledge is co-created via student–teacher collaboration [7]. Such an approach entails the teacher developing a new set of skills, including revoicing students' thinking by way of repeating, expanding, recasting, or translating student explanations for both the speaker and the rest of the class [16]. The teacher engages in hermeneutic listening [5], which Davis [24] describes as an intention to reflect the negotiated and participatory nature of this manner of interacting with learners. ... This sort of listening is an imaginative participation in the formation and the transformation of experience. Hermeneutic listening demands the willingness to interrogate the taken for granted and the prejudices that frame our perceptions and actions. (pp. 369-370)

Another teacher function is to provide and clarify keywords, such as: explain, justify, agree, compare, represent, and validate [5]. By providing such support, the teacher reduces extraneous cognitive load for the students, allowing them to focus working memory on the intrinsic cognitive load of the task [25]. In addition, the teacher models appropriate language and behaviours [26], including appropriate social interactions.

By far the greatest change in teacher behaviour in a MTLC is in the area of questioning.

Rather than being the primary questioner, the teacher moves to a secondary role, utilizing questions in response to student statements [26], and employing probing questions to move the student discussion forward [7]. Through questioning, the teacher scaffolds students' learning experience and provides guidance and direction for student thinking.

3. Case Study: A Whole-School Implementation of a MTLC

This case study examines how a large elementary school in Ontario, Canada implemented a MTLC across the school. More than 1,000 students attend the Kindergarten to Grade 5 school, located in a city whose population is approximately 500,000. The school population in turn is very diverse, with a high number of transient students, a large English as a second language component, and a large cohort of special needs students.

3.1 Methodology

A case study approach was used to examine how the tenets of a MTLC were instilled across Grades 1 through 5 classes. Collection of classroom artifacts was followed by 1-hour semi-structured interviews with teacher representatives of each grade team. The transcribed interview notes were then distributed to participants for member checking. Previous teacher professional learning sessions related to MTLCs were examined and linked to teacher actions in their classrooms. Triangulation of these sources provided internal validity to the study. A case study protocol [27] supported external validity, as did previous research on MTLCs [7-8, 20].

3.2 Research Questions

The study was framed around the following questions:

(1) How can a large elementary school implement a MTLC in a consistent and coherent manner across the entire school?

(2) What issues with respect to teacher training were encountered? How were these issues addressed?

(3) What implementation issues were encountered? How were these issues addressed?

(4) What, if at all, was the impact on student motivation and engagement?

3.3 Teacher Training and Professional Learning

The teachers at this school had attended several professional learning sessions describing a MTLC, as well as related concepts such as three-part lesson planning. One of these sessions (a full-day professional development unit) was devoted to implementation of a MTLC and involved an examination of the research behind a MTLC [7, 28] and possible implementation issues. Extensive group learning and role playing was employed to support teachers' deep learning around a MTLC. When the decision was made to implement a MTLC across the school, the principal purchased copies of Smith and Stein's [29] 5 Practices for Orchestrating Productive Mathematics Discussions, which teachers used as a basis for MTLC implementation. The approach focused on five practices:

(1) Anticipating likely student responses to challenging mathematical tasks;

(2) Monitoring students' actual responses to the tasks (while students work on the tasks in pairs or small groups);

(3) Selecting particular students to present their mathematical work during the whole-class discussion;

(4) Sequencing the student responses that will be displayed in a specific order;

(5) Connecting different students' responses and connecting the responses to key mathematical ideas. [29, p. 8].

When some teachers expressed a desire to learn more about different methods of presenting student solutions, the principal provided copies of Communication in the Mathematics Classroom [30] and arranged for an outside expert to provide professional learning support. This allowed teachers to become familiar with three presentation formats: gallery walk, bansho (board writing), and math congress. At the implementation stage, teachers appeared comfortable in using all three formats, depending on the task and mathematical content involved. Bruce [28] identifies five strategies for encouraging high quality student interaction:

(1) The use of rich math tasks

(2) Justification of solutions

(3) Students questioning one another

(4) Use of wait time

(5) Use of guidelines for math talk. (pp. 2-3)

Bruce [28] also enumerates six guidelines for math talk, with sentence starters:

(1) Explain: "This is my solution/strategy…", "I think ____ is saying that…"

(2) Agree with reason: "I agree because…"

(3) Disagree with reason: "I disagree because…"

(4) Build on: "I would like to build on that idea…"

(5) Go beyond: "This makes me think about …"; "Another way to think about this is…"

(6) Wait time. (p. 3)

The teachers in this case study made extensive use of these strategies and guidelines, and all involved felt that they grew professionally as they continued to embed MTLC into their classrooms.

Table 1 outlines sample MTLC tasks for each grade level. Teachers indicated that while they did not employ the MTLC strategy in every class, the tenets of the MTLC were used throughout the school year. Figure 1 illustrates the wealth of different solution methods that the MTLC groups employed. During the consolidation using Bansho, the solutions were grouped by technique and group members presented their explanations and justifications for the selected technique. A whole class discussion considered extensions, such as different sums for the three digits, having more digits on the lock, or having four-digit combinations rather than three-digit combinations.

During this discussion, students would first brainstorm in their groups and then propose results. The teacher commented that this process was particularly important: If you didn't get it, I still want to know what strategy, because sometimes, you might be doing splitting in your head, and I want to know that. I want to know how complex your thinking is, or how simple it might be. But if you're coming to the right answer, I want to know how did you get there. And if I can find a way for you to do it maybe more efficiently, or something that maybe long term is better then that's beneficial to me, because then I can maybe put people into groups or pairs and that kind of thing. (Teacher A, Grade 3)

Figure 2 illustrates some sorting rules for the Grade 4 shapes activity. The teacher found that by allowing total freedom of choice in the students' sorting rules, the entire class was engaged and active:

So they sorted them and then they recorded on paper however they wanted to, so there were a lot of different ways—charts and numbers and drawings and whatever. Then after that we did a math congress; we always do a sharing circle thing with our answers and they come up and explain how they got it and all that. (Teacher B, Grade 4)

The group whose work is illustrated in Figure 3 shows how the MTLC task supports students in learning and utilizing mathematical language. This group sorted the shapes based on number of vertices, demonstrating a clear understanding of the concept of vertex. Finally, a Grade 2 teacher was thrilled with the variety of strategies the students employed:

While they were doing it, we had so many different strategies happening in the class. Some used the algorithm, some used making 10s, tallies. So it was neat the classes were doing the same thing; we saw in each class they were all using different strategies. (Teacher C, Grade 2)

Grade	Learning goal	Task
1	We are learning to count objects so we can show numbers and number relationships in a variety of ways.	Each student was given a random number of Smarties and asked to count them in an organized way. Ten frames, which had not been used previously, were available in the classroom but were not required to be used, although some students chose to use them.
2	We are representing data in an organized way and using our two-digit addition skills.	Find out the total number of pockets in the classroom and show how you got the answer.
3	We are learning that a sum can be arrived at from a number of different number combinations and we are using math language to explain our results.	Imagine that you have a bike lock with only the digits 0 through 9. The sum of the three numbers needed to open the lock always adds up to 10. Explore the different ways you can make a sum of 10 with the numbers on the lock.
4	We are learning to sort 2-dimensional and 3-dimensional shapes in a variety of ways using attributes of the shapes.	Volunteers sort shapes on the Smartboard and the class lists possible sorting attributes. Groups then sort physical shapes and look for multiple ways to sort them.
5	We are learning to organize and represent data from primary sources in a variety of ways.	Building on a science experiment, a student asked the question: "Do taller people have bigger lung capacity?" Students then made hypotheses and investigated using balloons that they blew up and measured circumferences, and students' heights.

Table 1 Sample MTLC Tasks by Grade Level.

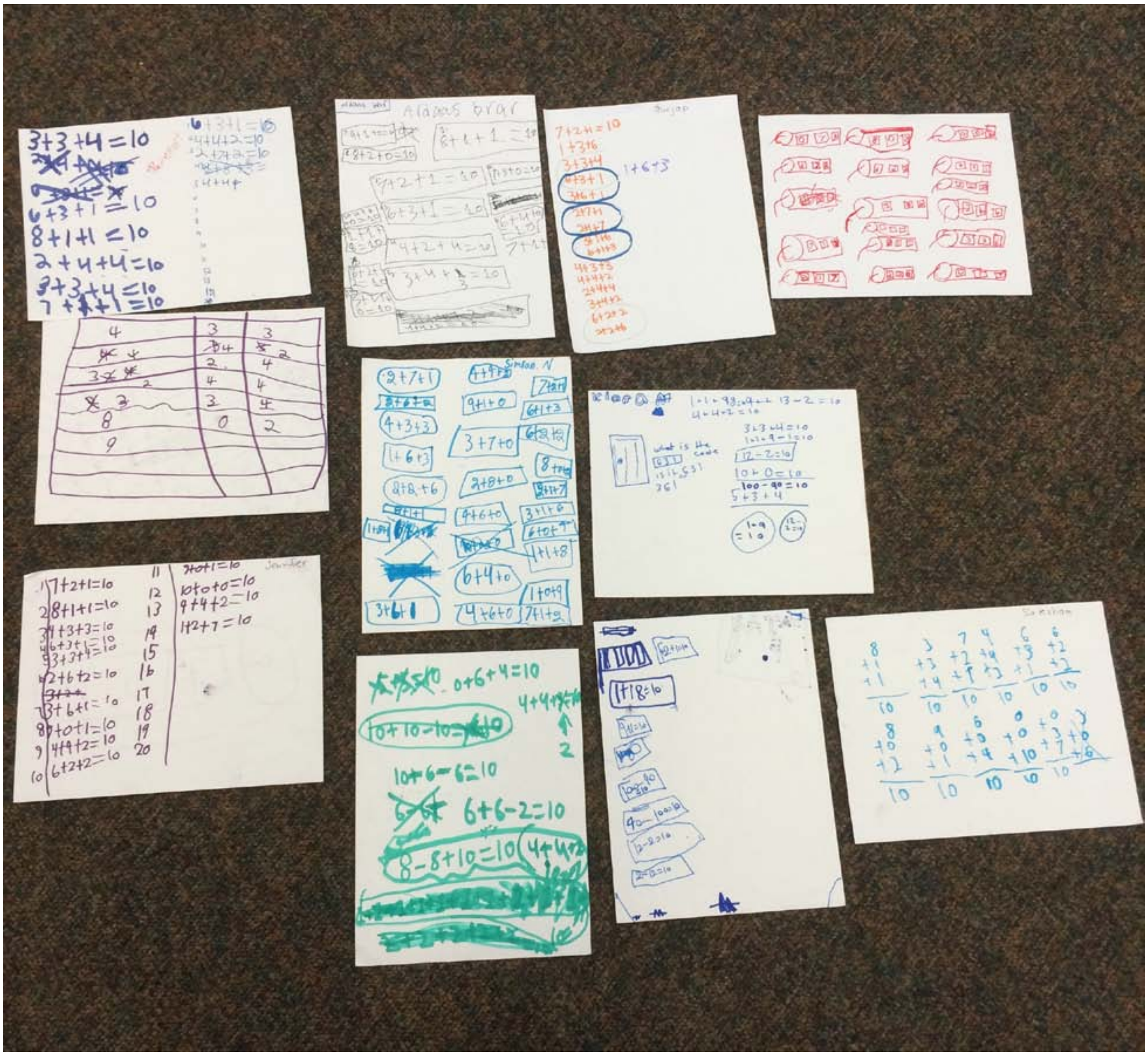

Fig. 1 Grade 3 bike lock solutions.

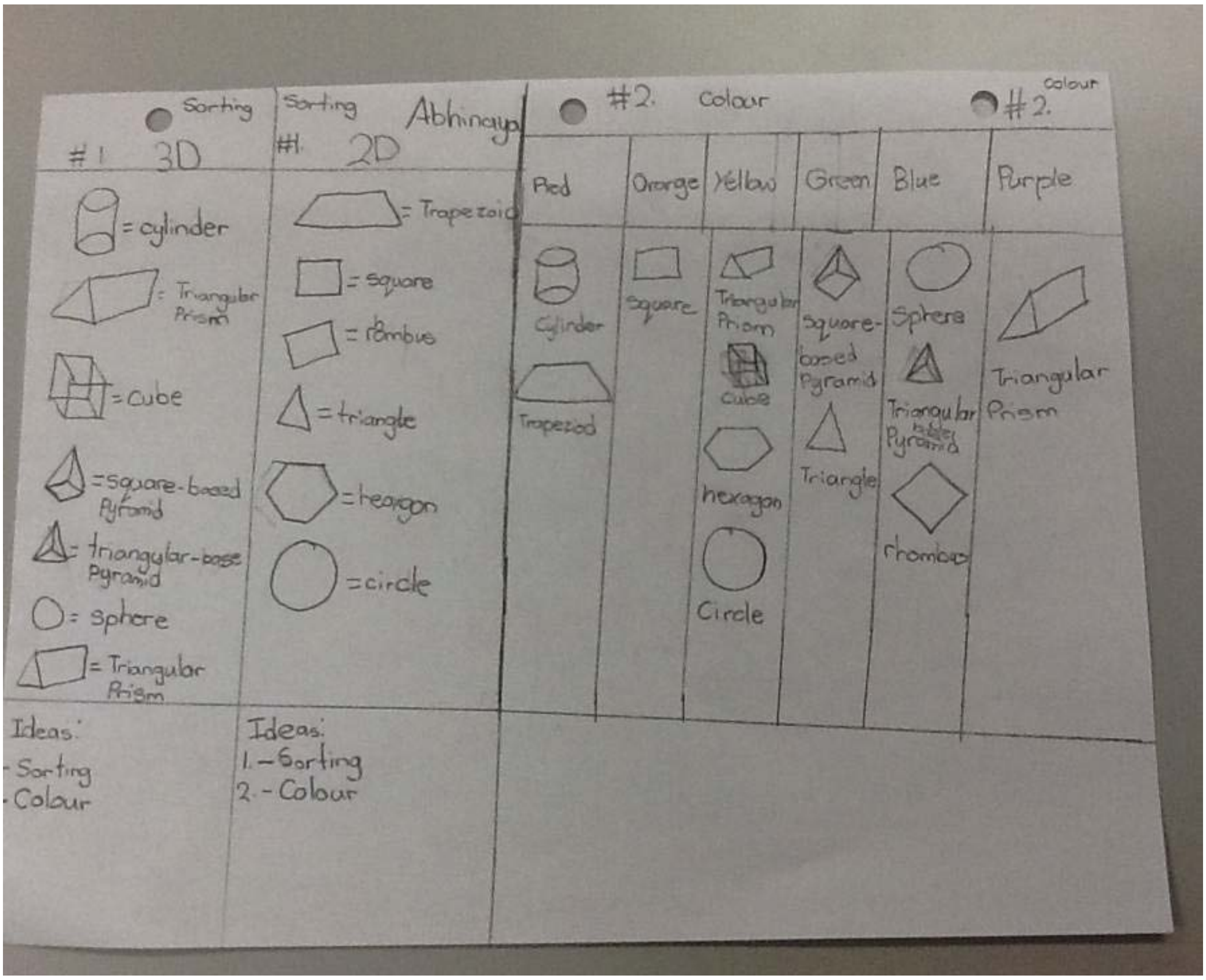

Fig. 2 **Grade 4 sorting by dimensions and by colour.**

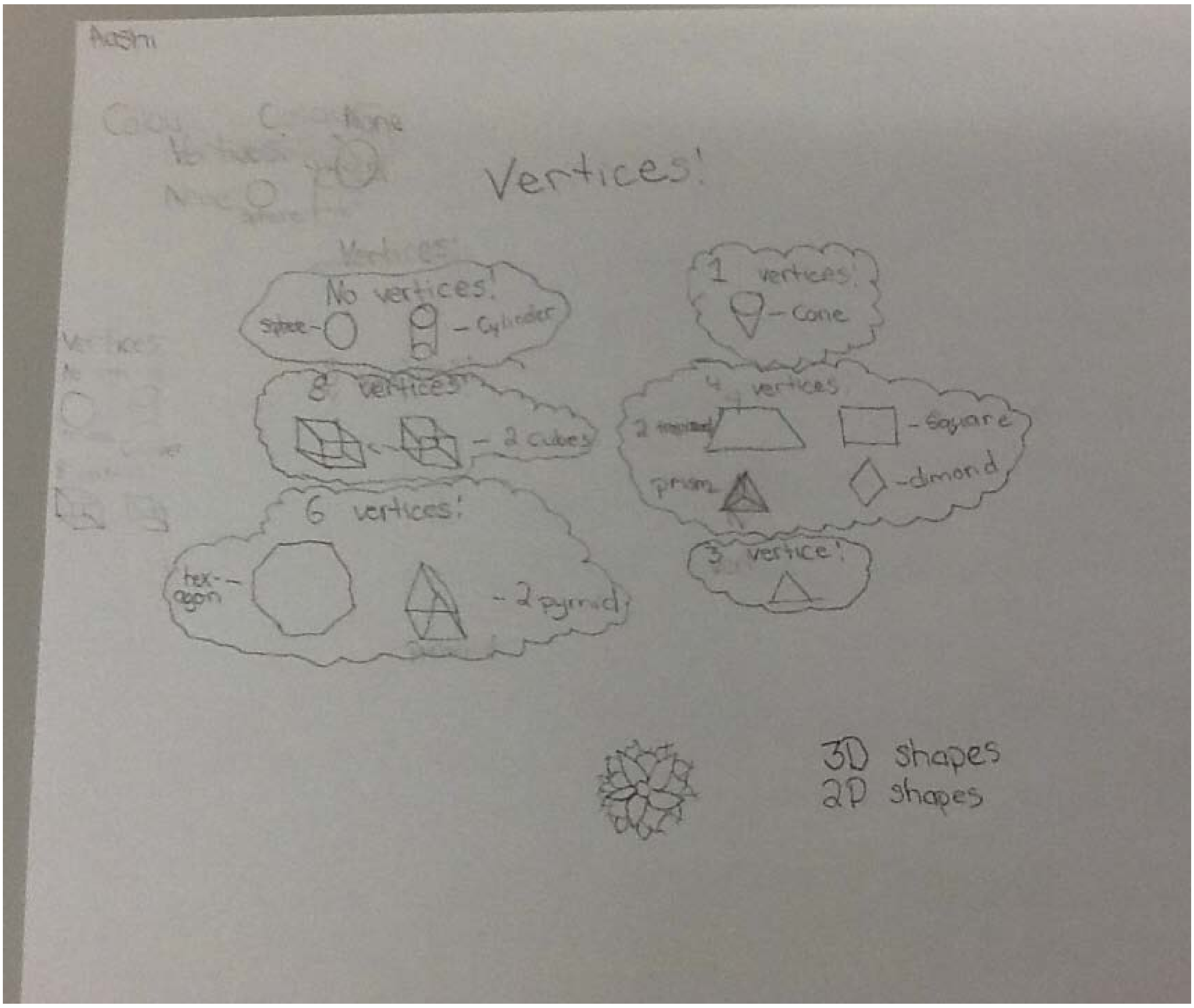

Fig. 3 Grade 4 Figure Sorting Using Mathematical Language.

3.4 Discussion

The Ontario curriculum policy documents specify that every grade will focus on seven Mathematical Processes: representing, connecting, communicating, problem solving, reasoning and proving, selecting tools and strategies, reflecting [31]. MTLCs are particularly efficient for addressing these processes. A typical task in a MTLC is a problem-solving task requiring students to represent information, connect information, select tools and strategies, communicate with their workmates and with others in the class as they explain their solutions, reason and prove or justify, and reflect on their work. The reflection phase frequently leads to extensions and conjectures. A Grade 5 teacher shared an anecdote regarding one of her students who, upon reflecting on the balloon experiment, stated:

This experiment in the first place is really flawed, because some of the kids can't blow up balloons, and other kids, the air might have leaked out of the balloon before they tied it, so I don't think this is really a well-designed experiment. (Student, Grade 5, cited by Teacher A)

With respect to research question 1—How can a large elementary school implement a MTLC in a consistent and coherent manner across the entire school?—one issue that was addressed early in the process was consistence and coherence. Whole-school meetings, with at least one representative from each grade team, identified appropriate levels of expectations around student thinking. Table 2 illustrates the whole-school coordination and student learning trajectories. Across all grades, students were expected to explain their thinking: In Grade 1, an additional focus wason agree or disagree with reasons; in Grade 2, the focus was on compare and contrast, identifying similarities and differences; Grades 3 and 4 emphasized justifying solutions and ways to validate solutions; and Grade 5 progressed to predicting and hypothesizing. The teachers also varied the sharing strategies, using a combination of gallery walk, Bansho, and math congress. Thus by Grade 5, the students had been taught the various sharing strategies and were comfortable with them.

Grade	Lesson	Sharing strategy	Keywords
1	Counting with Smarties	Gallery Walk	– Explain – Explain/disagree with reasons
2	How many pockets	Gallery Walk	– Explain – Compare
3	Bike lock patterns	Bansho	– Explain – Justify – Extend
4	Sort 2D & 3D shapes	Gallery Walk & Math Congress	– Explain – Justify
5	Lung capacity	Math Congress	– Explain – Justify – Predict – Hypothesize

Table 2 Summary of Lessons, Sharing Strategies, and Keywords

A second issue was the source of the tasks. Each grade team investigated possible tasks that could be appropriate for the mathematics content expectations they planned to address. Textbooks and the Internet provided a number of possibilities. In addition, some teachers had used rich tasks in previous years. The grade team narrowed the choices to probable tasks, and made modifications to the tasks if necessary. This process was time consuming but everyone involved commented on the usefulness of the activity. In many cases, multiple tasks were identified, and tasks not used during the current school year were archived for future use.

The potential problem of whole-school buy-in was not encountered. While the promotion of MTLC was a district-wide initiative, teachers as professionals make instructional decisions for their own classrooms. In discussing approaches to educational innovation and change, Hughes [15] points out that frequently across a whole school, some teachers will enthusiastically adopt a change, while others will opt for superficial compliance. In organizational behaviour research, teaching is classed as a professional bureaucracy. This means that while overall direction is set externally (by the state/province or by the district), teachers have much latitude in implementing policy at the school and classroom level [32]. To truly adopt change, teachers must be convinced of the utility of the change and its benefits for their students. Even then, complete implementation of an innovation frequently takes 10 years [15].

The school involved in this case study routinely employed distributed leadership in virtually every dimension of school life and culture. As a consequence, every teacher in the school bought in to the MTLC concept. Not every lesson involved MTLC, but the benefits to students of MTLC were clear to the teachers, and they plan to incrementally increase the MTLC lessons as they progress. Already, the concepts learned in the MTLC—such as explain, justify, select—are employed in their other classes. Although the initial direction was external, as opposed to traditional action research in which the impetus for change is internally generated, the teachers in this school adopted the MTLC strategy by employing steps similar to action research. The action research cycle involves planning, taking action, observing the results, evaluating the results, and modifying [15, 33]. In this school, extensive planning and training preceded implementation. MTLC activities were done in each class; other teachers in the grade team observed the implementation whenever possible, and grade teams discussed the outcomes and suggested modifications that were implemented in other classes in the grade.

With respect to research question 2—What issues with respect to teacher training were encountered? How were these issues addressed?—teachers met frequently prior to implementing the MTLC. They used a book study approach based on Smith and Stein's [29] text to build their capacity, and employed methods discussed by Bruce [28] to gain confidence in implementing a MTLC. One issue that arose was teacher self-efficacy, which Ross [34] found to be very resilient and difficult to change. In this case study, the concern was not mathematical content knowledge but rather the implementation of a pedagogical strategy that was new and unfamiliar to most of the teachers. By working in grade teams and consulting the expert provided by the principal, most teachers felt comfortable trying the MTLC in their classrooms. The teachers felt supported by their grade team and any pedagogical issues that arose were discussed with the team members and sometimes referred to the expert for suggestions.

In responding to research question 3—What implementation issues were encountered? How were these issues addressed?—the teachers identified several concerns. First, implementing the MTLC involved significantly more time than for more traditional lessons. Some teachers noted that the time allotted to the MTLC might prevent them from completing all curriculum expectations for the grade by the end of the school year. Interestingly, one teacher disagreed:

I find you cover a lot more of the curriculum, you know, graphing in there, double digit addition in there, and they were working with a partner, so they had to do a lot more interpersonal; they had to talk, they were presenting when we did the math talk, and then they were wandering around doing a gallery walk. This is so much that you can get done as a teacher, you observe so much, you can cover so much more. This year I feel way less stressed, always I'm like, "oh my gosh, I'm never going to finish the math," and this year I'm like "wow, I'm ahead of where I was last year." (Teacher B, Grade 2)

On a related note, several teachers pointed out the opportunities for cross-curricular tasks; for example, teachers mentioned using literacy-based tasks that involve mathematical ideas. The school had a wide array of resources in this area and grade teams utilized these resources to construct cross-curricular math tasks. The Grade 5 sample task demonstrated integration with the science curriculum, and the grade team identified other possible tasks linking the mathematics and science curriculum expectations. These approaches mitigate to some extent the time requirements of a MTLC. Some teachers also felt that the deeper learning through MTLC also allowed curriculum coverage at a faster rate.

Another implementation issue involved modifications and accommodations for students with special needs or English-as-a-second-language requirements. For the most part, these were non-issues since teachers in the school had regularly addressed such students' needs in past school years and were familiar with strategies to meet their needs. Frequently these modifications involved creating parallel tasks—for instance, the teacher poses two different questions at different levels but connected in terms of the big idea to which they relate and their context [35]. The tasks must be designed to allow all students to participate in the consolidation phase and math talk; for example, special needs students in Grade 2 were asked to find the total number of pockets in their group rather than in the whole class, while in Grade 4 the number and type of shapes that students were asked to sort was reduced.

Because social skills and interpersonal skills feature prominently in a MTLC, all grade levels explicitly taught these skills early in the school year. The meaning of keywords such as explain, justify, and extend (build on) were discussed before any MTLC activities were undertaken. In most classrooms, posters showing these keywords (along with their meaning and examples) were placed on classroom walls so that students could refer to them when needed. In addition, appropriate mathematics language was taught or reviewed when necessary.

Teachers referred to the need for extensive reflection-in-action [36-37] during MTLC tasks. A MTLC will result in multiple instances of students employing strategies and arguments that may be unexpected for their teachers. Teachers need to be able to understand, reflect on, and act on these student strategies, frequently leading to modifications of the planned lesson trajectory. Teachers' ability to reflect-in-action is related to their self-efficacy and their self-confidence. The majority of teachers in this case study indicated that they not only were comfortable with this process but also found it exciting and exhilarating.

Finally, with respect to research question 4—What, if any, was the impact on student motivation and engagement?—all teachers reported that students were engaged in the tasks, and motivated to ensure that all members of their group understood the strategies and participated in their solutions. This is consistent with research by Boaler [1] who found that students in a MTLC attempted to motivate each other when some students appeared unmotivated. Boaler also found that participating in a MTLC caused students to feel more confident and positive about mathematics. This is consistent with Jansen's [20] findings that linked student motivation to their beliefs about mathematics. Students in a MTLC learn that mathematics is co-created by the teacher and themselves, so student beliefs about mathematics are positively affected. Brown [5] found that for conditions such as a MTLC, "student engagement in building on each other's ideas can be fostered by providing learning environments that problematize subject matter, that share authority with students, that hold students accountable to disciplinary norms, and that provide students with relevant resources" (p. 117). In this case study, the Grade 1 team qualified their comments about student engagement, because they found that most students at that level were engaged in whatever tasks they were given, and the MTLC, while aligning with the conditions to foster engagement, had less impact than in the other grades. Additionally, teachers commented on the persistence fostered in a MTLC and noted that the shared responsibility resulted in far higher levels of persistence when confronted with an obstacle.

Surprisingly, although teachers had used the rubrics identifying progress toward a MTLC [7] during professional learning sessions prior to the school year, no reference was made to them during the implementation period. At an informal level, most teachers during interviews indicated where they believed they were situated in the progress toward a MTLC without reference to the rubrics; the teachers evaluated the success of the MTLC by reference to student learning. Teachers in the same grade frequently compared student work samples across classes and noted increased higher-order thinking compared to previous years. Another strategy that was employed was to give students sticky notes when doing gallery walks. Students were asked to comment on posted solutions, adding questions, comments, or identifying similarities across solutions. A variation on this approach involved asking students to sign the solution indicating understanding. This functioned as a self-evaluation strategy: “Children have to sign it if they understood it, if they can repeat what was done. If they don’t sign it, you know they couldn’t repeat the information” (Teacher B, Grade 2)

4. Limitations

A number of limitations affect the replication of the MTLC described in this case study. First, the school selected for this study was not chosen randomly; it had a reputation for innovation, particularly involving technology. It also had a well-trained and enthusiastic teaching staff, who had both high mathematical self-efficacy as well as a history of motivating students and modeling positive attitudes towards mathematics. The teachers routinely participated in training and worked to grow their professional practice as teachers.

Second, the principal of the school was very experienced and had a reputation as one of the best principals in the district. She regularly employed a distributed leadership approach in almost all aspects of school decision making. As a consequence, the teaching staff was comfortable with taking the initiative in new directions and implementing change. Staff meetings were routinely differentiated based on teacher knowledge levels and interests, so that teachers chose which sessions to attend to increase their professional knowledge.

Third, the principal was willing to commit resources to her teachers’ training, including texts and other print materials, arranging professional learning sessions, and bringing in experts to support implementation. In addition, the grade teams functioned as communities of practice, generating strategies and evaluating outcomes with a view to improvement.

All these conditions resulted in an efficient and effective implementation of a MTLC across the entire school. Replicating this level of success would require having all such conditions in place in another location.

5. Conclusion

The goal of any educational change is to increase student achievement. Extensive research has shown that MTLCs result in deeper learning, improved motivation and engagement, significant improvements in students’ interpersonal skills, and increased student understanding and achievement. The case study described in this paper identifies a method for successfully scaling up MTLC from the classroom level to the whole-school level. Subject to the limitations outlined above, this implementation model may be replicable across schools although again in this case the school culture and distributed mitigated the MTLC’s successful innovation. Not all schools mirror such attributes, and thus wider implementation may be more difficult in some instances. Nonetheless, MTLCs’ benefits for students outweigh the possible difficulties and thus MTLCs should be considered by individual schools and school districts.

References

[1] J. Boaler, Promoting “relational equity” and high mathematics achievement through an innovative mixed-ability approach, British Educational Research Journal 34 (2) (2008) 167-194. doi: 10.1080/01411920701532145.

[2] C. Kapellidi, The interplay between agency and constraint: Some departures from the organization of talk in the classroom, Text & Talk 35 (4) (2015) 453-479, doi: 10.1515/text-2015-0012.

[3] J. Sutherland, Going “meta”: Using a metadiscoursal approach to develop secondary students’ dialogic talk in small groups, Research Papers in Education 30 (1) (2015) 44-69. doi:10.1080/02671522.2013.850528.

[4] L. Bofferding, M. Kemmerle, Elementary teacher candidates’ use of number strings: Creating a math-talk learning community. Mathematics Teacher Educator 3 (2) (2015) 99-115.

[5] R. Brown, Exploring the social positions that students construct within a classroom community of practice, International Journal of Educational Research 46 (3) 116-128, doi:10.1016/j.ijer.2007.09.012.

[6] R. Gillies, A. Khan, Promoting reasoned argumentation, problem-solving and learning during small-group work.

Cambridge Journal of Education 39 (1) (2009) 7-27, doi: 10.1080/03057640802701945.

[7] K. Hufford-Accles, K. Fuson, M. Gamoran Sherin, Describing levels and components of a math-talk learning community, Journal for Research in Mathematics Education 35 (2) (2004) 81-116.

[8] S.S. McCrone, The development of mathematical discussions: An investigation in a fifth-grade classroom, Mathematical Thinking and Learning 7 (2) (2005) 111-133, doi: 10.1207/s15327833mtl0702_2.

[9] J. Hunter, Developing a productive discourse community in the mathematics classroom. The New Zealand Mathematics Magazine 46 (1) (2009) 1-12.

[10] C. Murphy, Changing teachers' practices through exploratory talk in mathematics: A discursive pedagogical perspective, Australian Journal of Teacher Education, 40 (5) (2015) 61-84, doi: 10.14221/ajte.2015v40n5.4.

[11] T. Wood, Creating a context for argument in mathematics class. Journal for Research in Mathematics Education 30 (2) (1999) 171-191. doi: 10.2307/749609.

[12] B. Davis, D. Sumara, Constructivist discourses and the field of education: Problems and possibilities. Educational Theory 52 (4) (2002) 409-428, doi: 10.1111/j.1741-5446.2002.00409.x.

[13] H. Bauersfeld, The structuring of the structures: Development and function of mathematizing as a social practice. In L.P. Steffe & J. Gale (Eds.), Constructivism in education (pp. 137-158). Mahwah, NJ: Lawrence Erlbaum, 1995.

[14] K. Gergen, Social construction and the educational process. In C. Fosnot (Ed.), Constructivism: Theory, perspectives, and practice (pp. 117-140). New York, NY: Teachers College Press, 2005.

[15] G. Hughes, Ipsative assessment: Motivation through marking progress, London, UK: Palgrave Macmillan, 2014.

[16] E. Forman, E. Ansell, The multiple voices of a mathematics classroom community. Educational Studies in Mathematics 46 (1) (2001) 115-142, doi: 10.1023/A:1014097600732

[17] B. Davis, E. Simmt, Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. Educational Studies in Mathematics 61 (2006) 293-319, doi:10.1007/s10649-006-2372-4

[18] R. Wegeril, N. Mercer, Language for thinking: A study of children solving reasoning test problems together. In H. Cowie & G. van der Aalsvoort (Eds.), Social interaction in learning and instruction: The meaning of discourse for the construction of knowledge, (pp. 179-192). Oxford, UK: Elsevier Science, 2000.

[19] R. Brown, P. Renshaw, Collective argumentation: A sociocultural approach to reframing classroom teaching and learning. In H. Cowie & G. van der Aalsvoort (Eds.), Social interaction in learning and instruction: The meaning of discourse for the construction of knowledge (pp. 52-66). Oxford, UK: Elsevier Science, 2000.

[20] A. Jansen, Seventh graders' motivations for participating in two discussion-oriented mathematics classrooms, The Elementary School Journal 106 (5) (2006) 409-428, doi: 10.1086/505438

[21] P. DeAnda, Weather and Russian dolls: Students negotiating what it means to do math. In T. Lamberg & L. R. Wiest (Eds.), Proceedings of the 29th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education (pp. 715-718). Lake Tahoe, NV: University of Nevada, Reno, 2007.

[22] P. Naylor, & H. Cowie, Learning the communication skills and social processes of peer support: A case study of good practice. In H. Cowie & G. van der Aalsvoort (Eds.), Social interaction in learning and instruction: The meaning of discourse for the construction of knowledge (pp. 93-103), Oxford, UK: Elsevier Science, 2000.

[23] R. Gillies, K. Nichols, G. Burgh, M. Haynes, The effects of two strategies and meta-cognitive questioning approaches on children's explanatory behaviour, problem-solving, and learning during cooperative, inquiry-based science, International Journal of Educational Research 53 (2012) 93-106, doi: 10.1016/j.ijer.2012.02.003

[24] B. Davis, Listening for differences: An evolving conception of mathematics teaching. Journal for Research in Mathematics Education 28 (3) (1997) 355-376, doi: 10.2307/749785

[25] J. Irvine, Fueling the mind: Maximizing student learning and performance. Ontario Principals Council Register (2016, February) 6-14.

[26] R. Gillies, M. Haynes, Increasing explanatory behaviour, problem-solving, and reasoning within classes using cooperative group work, Instructional Sciences 39 (3) (2011) 349-366, doi: 10.1007/s11251-010-9130-9

[27] R. Yin, Case study research: Design and methods (4th ed.). Thousand Oaks, CA: Sage, 2009.

[28] C. Bruce, Student interaction in the math classroom: Stealing ideas or building understanding. What Works? Research into Practice, 2009. Retrieved from

http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Bruce.pdf

[29] M. Smith, M. Stein, 5 practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics, 2011.

[30] Literacy and Numeracy Secretariat, Communication in the mathematics classroom (Capacity building series, no. 13), Toronto, ON: Queen’s Printer for Ontario, 2010.

[31] Ontario Ministry of Education, The Ontario curriculum, grades 1 to 8—Mathematics (Revised). Toronto, ON: Queen’s Printer for Ontario, 2005. Retrieved from http://www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf

[32] H. Mintzberg, The structuring of organizations, Englewood Cliffs, NJ: Prentice Hall, 1979.

[33] A. Johnson, A short guide to action research, Boston MA: Pearson, 2005.

[34] J. Ross, The stability and resilience of mathematics teacher self- efficacy. Paper presented at the Fields Mathematical Education Forum, Peterborough, ON, 2009.

[35] M. Small, Differentiating math instruction K-8. Toronto, ON: Nelson, 2009.

[36] R. Lane, H. McMaster, J. Adnum, & M. Cavanagh, Quality reflective practice in teacher education: A journey towards shared understanding. Reflective Practice 15 (4) (2014) 481-494, doi:10.1080/14623943.2014.900022

[37] D. Schön, The reflective practitioner, New York, NY: Basic Books, 1983.